Household Risk Management
and Optimal Mortgage Choice

John Y. Campbell

João F. Cocco

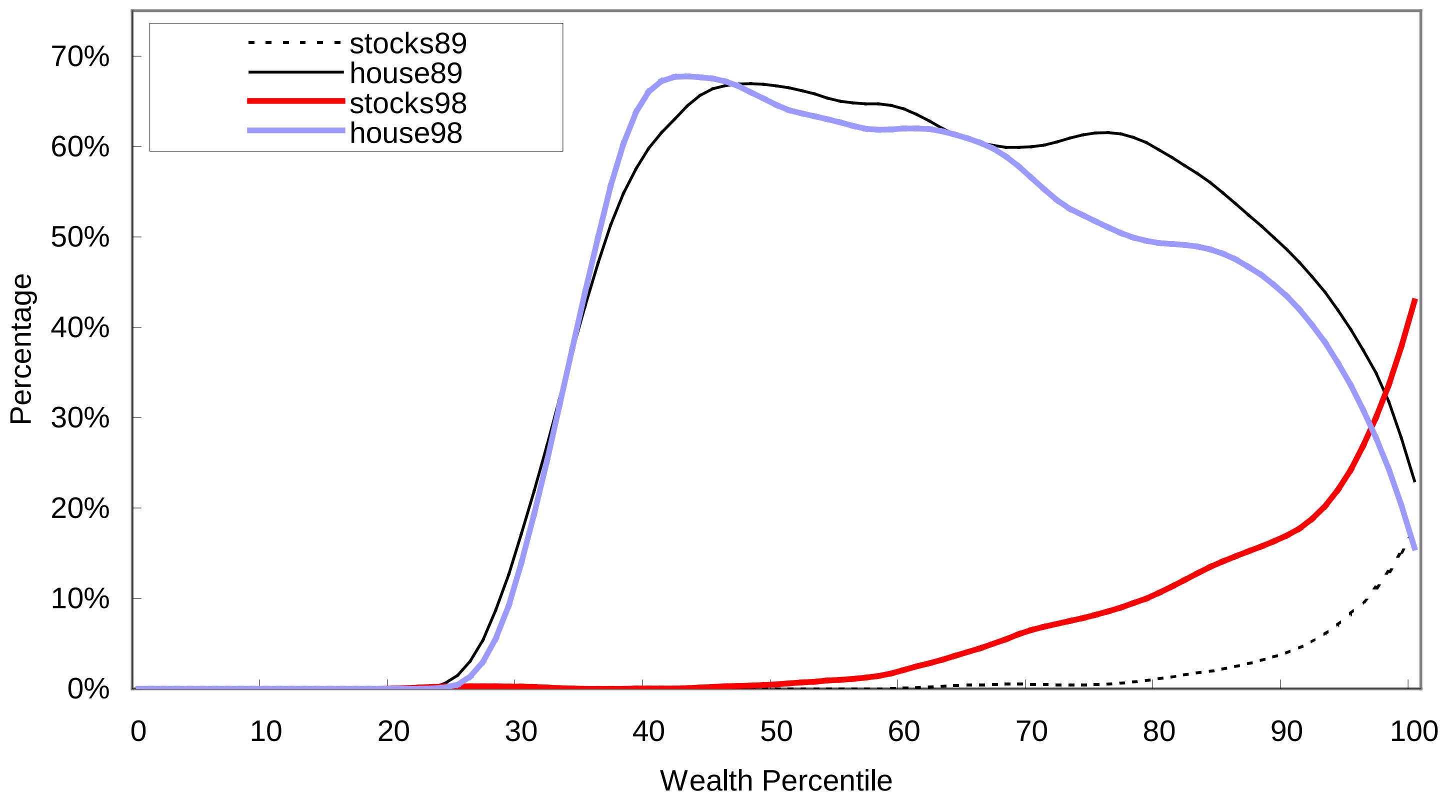
The 2nd Annual Conference on Personal Risk Management

Thursday 21st November 2002

Introduction

- Typical household portfolios are not diversified, unconstrained portfolios of liquid assets.
  - Major asset is an illiquid house.
  - Homeownership financed through a mortgage contract.
  - Borrowing constraints are important.

Figure 1: Portion of Household Assets in Corporate Equity and Real Estate by Wealth Percentile, 1989 - 1998



- The form of the mortgage contract can have large effects on the risks faced by homeowners.
- Mortgage contracts are often complex and differ along many dimensions. But two main categories:
  - Adjustable-rate (ARM) and fixed-rate mortgages (FRM).
  - We characterize the advantages and disadvantages of each type for different households.
  - We compare nominal and inflation-indexed FRM's.

- When deciding on the type of mortgage, an extremely important consideration is labor income and the risk associated with it.
- We solve a dynamic model of the optimal consumption and mortgage choices of a finitely lived investor who is endowed with non-tradable human capital that produces a risky stream of labor income.
  - Framework: the buffer-stock savings model of Zeldes (1989), Deaton (1991), and Carroll (1997).
  - The investor initially buys a house with a required minimum downpayment, financing the rest of the purchase with either an ARM or an FRM.
  - Subsequently the investor can refinance the FRM if it is optimal to do so.
  - We also allow the investor to take out a second loan against any housing equity in excess of the minimum downpayment.
  - In each period there is a fixed probability that the investor will move house.

Our results illustrate a basic tradeoff between two types of risk:

1. A nominal FRM, without a prepayment option, is a highly risky contract because its real capital value is highly sensitive to inflation.
2. A prepayment option protects the homeowner against one side of this risk. But this option raises the interest rate on an FRM and leaves the homeowner with a contract that is normally expensive, but extremely cheap if inflation is high.
3. An ARM is a safe contract in the sense that its real capital value is almost unaffected by inflation.

4. The risk of an ARM is the risk of short-term variability in real interest payments. This variability would not matter if there were free borrowing against future income, but it does matter if the homeowner faces binding borrowing constraints.

   - Constraints bind in states of the world with low income and low house prices.
   - The danger of an ARM is that it will require higher interest payments in this situation, forcing a temporary but unpleasant reduction of consumption.
   - Households with large houses relative to their income, volatile labor income, or high risk aversion are particularly adversely affected by this ARM risk.

5. The mobility of a household also affects the form of the optimal mortgage contract: If a homeowner knows he is highly likely to move in the near future, he is more likely to use the kind of mortgage that has the lower current interest rate.

   - Unconditionally, this is the ARM.
   - But it might be the FRM if the short-term interest rate is currently high and likely to fall.

- There is a large literature on mortgage choice:
  - FRM prepayment behavior and its implications for the pricing of mortgage-backed securities (Schwartz and Torous 1989 and Stanton 1995).
  - Households know more about their moving probabilities than lenders do; this creates an adverse selection problem in prepayment (Dunn and Spatt 1985, Chari and Jagannathan 1989, Brueckner 1994, LeRoy 1996, Stanton and Wallace 1998).
  - Alm and Follain (1984) emphasize the importance of labor income and borrowing constraints for mortgage choice, but their model is deterministic and thus they cannot address the risk management issues that are the subject of this paper.
  - Stanton and Wallace (1999) discuss the interest-rate risk of ARMs, but without considering the role of risky labor income and borrowing constraints.
- We are not aware of any previous theoretical work that treats income risk and interest-rate risk within an integrated framework as we do here.

A Model of Mortgage Choice

Time parameters and preferences

- We model the consumption and asset choices of a household with a time horizon of T periods.
- We study the decision of how to finance the purchase of a house of a given size $\overline{H}$.
- In each period t, $t = 1, ..., T$, the household chooses consumption of all goods other than housing, C_t. The date t nominal price of consumption is denoted by P_t.
- We assume preference separability between housing and consumption.
- The objective function of the household:

$$\max_{C_t} E_0 \sum_{t=0}^{T} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} + \beta^{T+1} \frac{W_{T+1}^{1-\gamma}}{1-\gamma}, \tag{1}$$

β is the time discount factor

γ is the coefficient of relative risk aversion.

W_{T+1} is terminal real wealth

The term structure of nominal and real interest rates

- We assume that one-period expected inflation follows an AR(1) process,

$$\pi_{1t} = \mu(1-\phi) + \phi\pi_{1,t-1} + \epsilon_t, \tag{2}$$

  where ϵ_t is a normally distributed white noise shock with mean zero and variance σ_ϵ^2.

- The one-period expected real interest rate is given by

$$r_{1t} = \overline{r} + \psi_t, \tag{3}$$

  where ψ_t is white noise.

- We assume away ex post inflation uncertainty, so the expected real interest rate equals the realized real interest rate. This assumption should have very little effect on our results, since short-term inflation uncertainty is modest and affects ARMs and FRMs symmetrically.

- The one-period nominal interest rate is

$$y_{1t} = r_{1t} + \pi_{1t} + \zeta. \tag{4}$$

- To model long-term nominal interest rates, we assume that the log pure expectations hypothesis holds:

$$y_{nt} = (1/n) \sum_{i=0}^{n-1} E_t[y_{1,t+i}]. \tag{5}$$

- This model implies that excess returns on long-term bonds over short-term bonds are unpredictable, even though changes in nominal short rates are partially predictable.

Available mortgage contracts

- At date 1, the household finances the purchase of a house of size $\overline{H}$ with a nominal loan of $(1-\lambda)P_1^H\overline{H}$, where λ is the required downpayment ratio.
- The mortgage loan is assumed to have maturity T, so that it is paid off by period $T+1$.
- If the household chooses a FRM, and the current interest rate on a FRM with maturity T is $Y_{T,1}^F$, then in each subsequent period the household must make a real mortgage payment, M_t^F, of:

$$M_t^F = \frac{(1-\lambda)P_1^H\overline{H}}{P_t \sum_{j=1}^{T}(1+Y_{T,1}^F)^{-j}}. \tag{6}$$

- We allow for a prepayment option. A household that chooses an FRM may in later periods refinance at a monetary cost of ρ. The indicator variable I_t^ρ equals one when the household refinances, zero otherwise.
- The date t nominal interest rate on a FRM is

$$Y_{T-t+1,t}^F = Y_{T-t+1,t} + \theta^F, \tag{7}$$

where θ^F is a constant mortgage premium.

- If the household chooses an ARM, the annual real mortgage payment, M_t^A, is given by:

$$M_t^A = \frac{Y_{1,t}^A D_t + \Delta D_{t+1}}{P_t}, \tag{8}$$

where ΔD_{t+1} is the component of the mortgage payment at date t that goes to pay down principal rather than pay interest. This is set equal to the principal paydown of an FRM.

- The date t nominal interest rate on an ARM is assumed to be equal to the short rate plus a constant premium:

$$Y_{1,t}^A = Y_{1,t} + \theta^A. \tag{9}$$

- In case the household cannot meet mortgage payments and is forced to default we assume that the household is left with a certain lower bound of lifetime utility.

- An inflation-indexed FRM with fixed real payments and real interest rate $R^I_{T,1}$ to maturity T requires a real mortgage payment of

$$M^I_t = \frac{(1-\lambda)P^H_1\overline{H}}{\sum_{j=1}^{20}(1+R^I_{T,1})^{-j}}.$$

- Real interest rate is given by

$$R^I_{T-t+1,t} = R_{T-t+1,t} + \theta^I.$$

- We also consider an inflation-indexed FRM with declining real payments and the same real interest rate.

Labor income risk

- The household is endowed with stochastic real labor income in each period, L_t, which cannot be traded or used as collateral for a loan.
- Household j's age t real labor income is exogenous and is given by:

$$l_{jt} = f(t, Z_{jt}) + v_{jt} + \omega_{jt}, \tag{10}$$

  where $f(t, Z_{jt})$ is a deterministic function of age t and other individual characteristics Z_{jt}, and v_{jt} and ω_{jt} are stochastic components of income.
- Transitory income ω_{jt} is IID normal with mean zero and variance σ_ω^2.
- Permanent income v_{jt} follows a random walk:

$$v_{jt} = v_{j,t-1} + \eta_{jt}, \tag{11}$$

  where η_{jt} is IID normal with mean zero and variance σ_η^2.

House prices and second loans

- Let p^H_{jt} denote the date t real log price of one unit of housing for household j. Real house price growth is given by:

$$\Delta p^H_{jt} = g + \delta_{jt}, \tag{12}$$

constant g plus an IID normal shock δ_{jt} with mean zero and variance σ^2_δ

- To save on state variables we assume that innovations to real house price growth are perfectly positively correlated with innovations to aggregate real labor income so that

$$\delta_{jt} = \alpha \eta_{jt}, \tag{13}$$

where $\alpha > 0$.

- We allow households who have accumulated housing equity to obtain a second one-period loan:

$$B_t \leq (1 - \lambda) P^H_t \overline{H} - D_t. \tag{14}$$

- The nominal interest rate on the second loan is equal to Y_{1t} plus a constant premium, θ^B

Taxation

- Linear tax at rate τ on gross labor income L_t.
- Mortgage interest is tax deductible.

Moving

- With probability p the household moves in each period
- The household sells the house, pays off the remaining mortgage, and evaluates utility of wealth using the terminal utility function.

Summary of the household's optimization problem

- The household's control variables are $\{C_t, B_t, I_t^{\rho}\}_{t=1}^{T}$.
- The vector of state variables is $X_t = \left\{t, y_{1t}, W_t, P_t, y_{1,t'}, t', v_t\right\}_{t=1}^{T}$.
- The equation describing the evolution of real cash-on-hand (when B_t is equal to zero and there is no refinancing at period t)

$$W_{t+1} = (W_t - C_t - (1-\tau)M_t)(1 + R_{1,t+1}) + (1-\tau)L_{t+1}. \tag{15}$$

Solution technique

- This problem cannot be solved analytically.
- We discretize the state space and the choice variables using equally spaced grids in the log scale.
- The density functions for the random variables were approximated using Gaussian quadrature methods to perform numerical integration (Tauchen and Hussey 1991).
- Given the finite nature of the problem a solution exists and can be obtained by backward induction.
  - To compute the continuation value for points which do not lie on the grid we use cubic spline interpolation.

Parameterization

Time, Preference, and Interest Rate Parameters

- We study the optimal consumption and mortgage choices of investors who buy a house early in life. That is, adult age in our model starts at age 26 and we let T be equal to 30 years.
- To keep the problem tractable, we set the time interval equal to 2 years. All parameters are stated on a 2-year basis.
- Baseline preference parameters: $\beta = 0.9604 = 0.98^2$ and $\gamma = 3$.
- We use CPI to measure inflation and 1-year Treasury return to measure nominal short rate, annually from 1962 to 1999.

Mortgage contracts

- Two important parameters of the mortgage contracts are the mortgage premiums, θ^F and θ^A.
  - It is natural to assume that $\theta^F \geq \theta^A$.
- To estimate the mortgage premium on FRM contracts we compute the difference between interest rates on commitments for fixed-rate 30 year mortgages and the yield to maturity on 30-year treasury bonds.
- The ARM interest rate data is from the monthly interest rate survey of the Federal Housing Finance Board (FHFB).

Table 1: Estimated parameters of the interest rate, labor income, and house price processes

Description	Parameter	Value
Mean log inflation	μ	.046
S.d. of log inflation	$\sigma(\pi_{1t})$	.039
Autoregression parameter	ϕ	.569
Mean log real yield	$\overline{r}$	.020
S.d. of real log yield	$\sigma(r_{1t})$	.022
S.d. of transitory income shocks	σ_ω	.141
S.d. of persistent income shocks	σ_η	.020
Mean log real house price growth	g	.009
S.d. of log real house price growth	σ_δ	.115
Corr. of house price shocks and inc. shocks		.027*

\Large* significant at the 2 percent level.

Table 2: Calibrated parameters

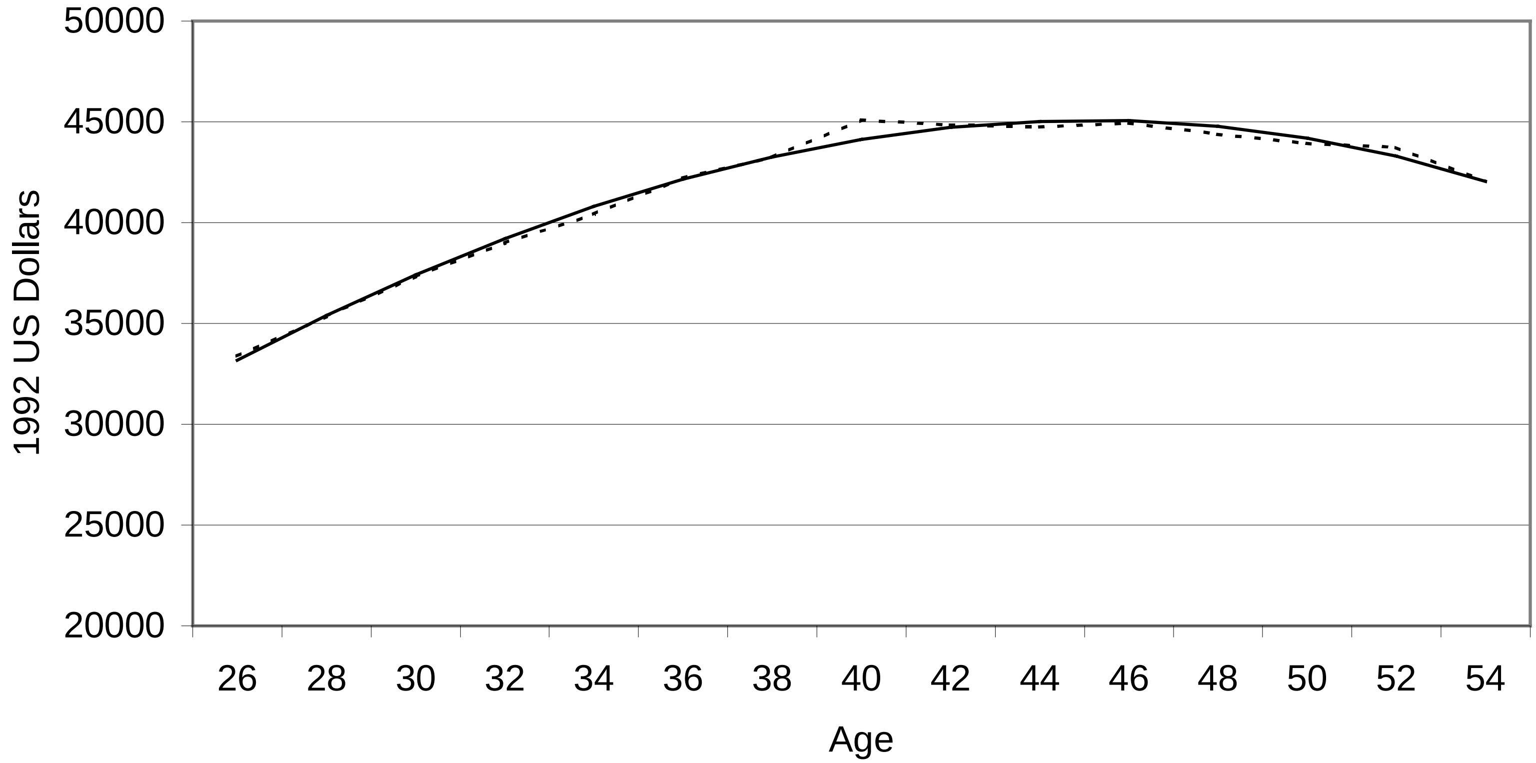
Description	Parameter	Value
Risk aversion	γ	3
Discount factor	β	.98
House size	$\overline{H}$	\$150,000
Downpayment	λ	0
Tax rate	τ	.20
FRM premium	θ^F	.016
Refinancing cost	ρ	\$1,000
ARM premium	θ^A	.010
Second loan premium	θ^B	∞

All parameters are in annual terms.

Labor income risk

- We use the family questionnaire of the Panel Study on Income Dynamics (PSID).
- We drop families that are part of the Survey of Economic Opportunities subsample.
- We use a broad definition of labor income.
- Labor income is deflated using the Consumer Price Index, with 1992 as the base year.
- The estimation controls for family-specific fixed effects.
  - $f(t, Z_{it})$ is assumed to be additively separable in t and Z_{it}.
  - The vector Z_{it} includes marital status, household composition, and the education of the head of the household.
  - The residuals obtained from the fixed-effects regressions of (log) labor income on $f(t, Z_{it})$ can be used to estimate σ_η^2 and σ_ω^2.
  - There probably is substantial measurement error in PSID data that biases upwards our estimate of the standard deviation of temporary income shocks. In the benchmark case we set it equal to a lower value of 0.20.

Figure 2: Two-year labor income profile.



House prices and second loans

- We use house price data from the PSID for the years 1970 through 1992.
- The self assessed value of the house was deflated using the Consumer Price Index, with 1992 as the base year, to obtain real house prices.
- We then correlate house prices with income at the household level.
- In the current draft we do not allow for a second loan.

Results

- We use our model to compare the welfare implications of fixed and adjustable rate mortgages.
  - We calculate lifetime expected utilities under alternative FRM and ARM contracts: the distribution of realized lifetime utility, based on simulation of the model across one thousand households.
  - The average welfare gain of the FRM relative to the ARM in the form of standard consumption-equivalent variations:
    * We calculate an average by weighting each state by its ergodic or steady-state probability.
    * For each mortgage contract we compute the constant consumption stream that makes the household as well off in expected utility terms.
    * Utility losses are then obtained by measuring the change in this equivalent consumption stream across mortgage contracts.

Figure 3: Benchmark utility distribution

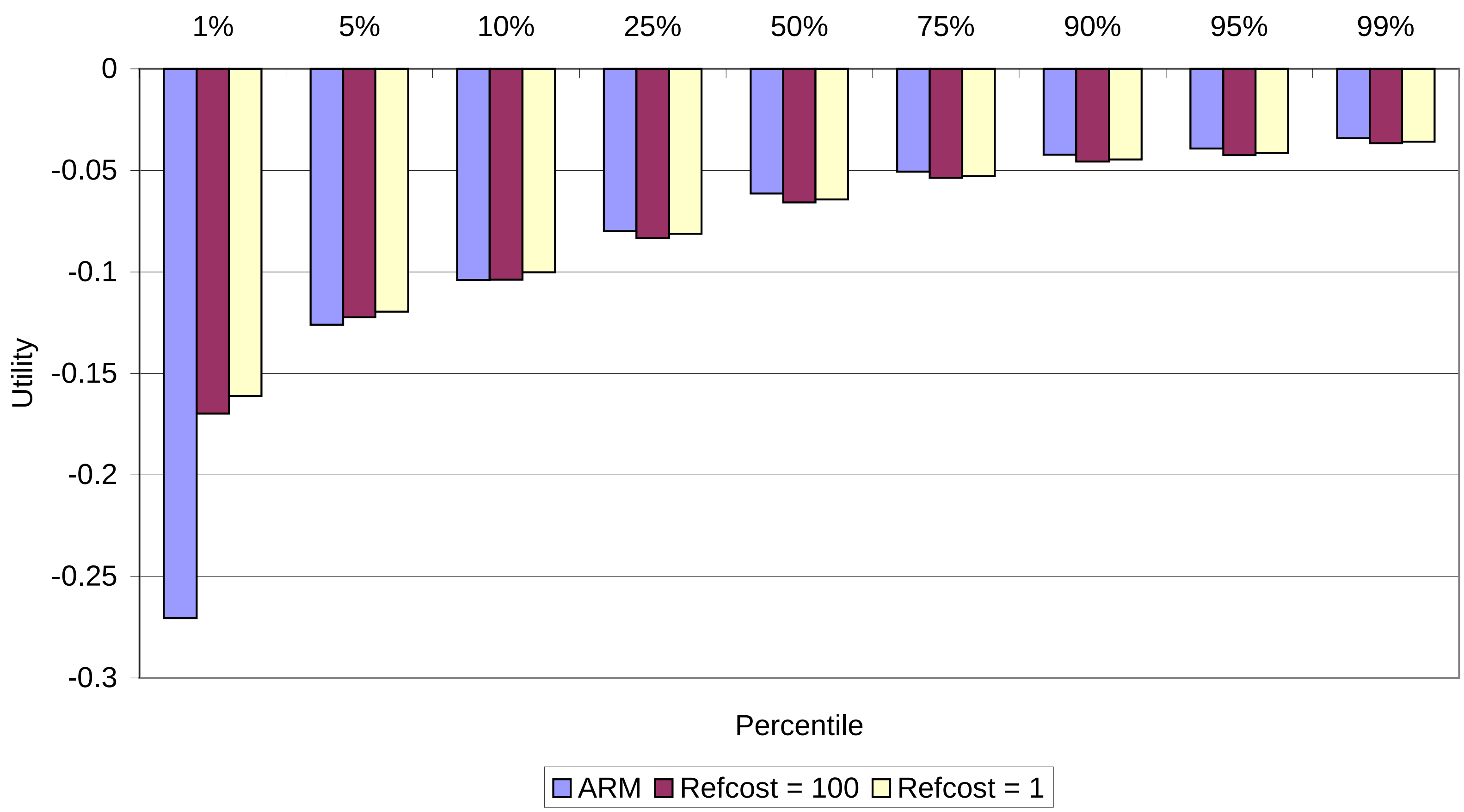


Table 3: Average welfare gains of nominal FRMs over ARMs

	FRM		Refinancing
	$\rho = \$1,000$	$\rho = \$100,000$	Option
$\gamma = 1/2$	-3.62%	-4.54%	0.92%
$\gamma = 3$	2.27%	1.00%	1.27%
$\gamma = 5$	34.63%	32.74%	1.89%
$\overline{H} = 100$	-2.88%	-3.45%	0.57%
$\overline{H} = 150$	2.27%	1.00%	1.27%
$\overline{H} = 200$	14.51%	11.32%	3.19%
$\sigma_\omega = 0.05$	-0.87%	-2.02%	1.15%
$\sigma_\omega = 0.20$	2.27%	1.00%	1.27%
$\sigma_\omega = 0.35$	15.47%	13.83%	1.65%

Figure 4: Utility distribution with nominal and inflation-indexed mortgage contracts

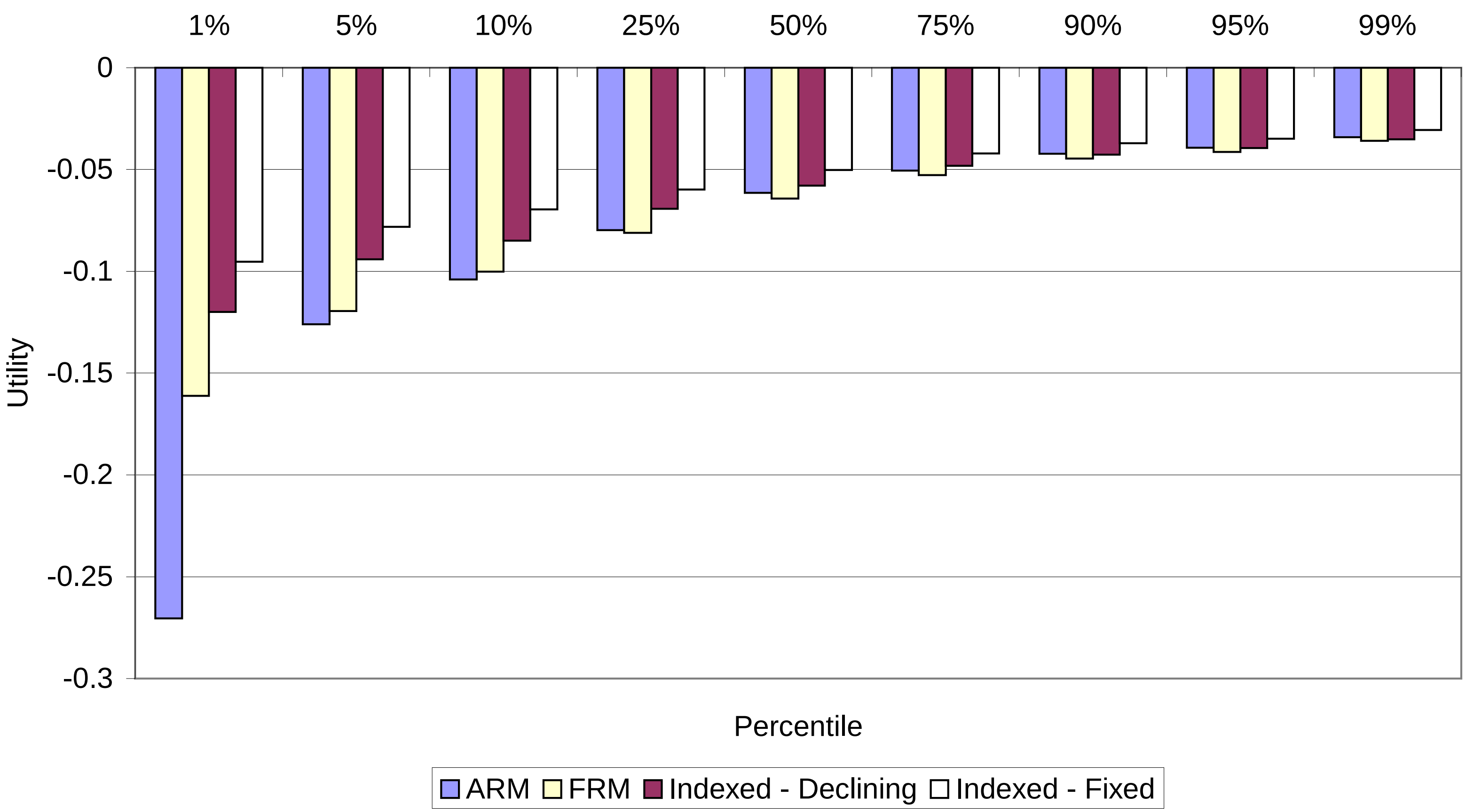


Table 6: Average welfare gains of nominal and inflation-indexed FRMs over ARMs

	Nominal FRM	Inflation-Indexed FRM Constant	Inflation-Indexed FRM Declining
$\gamma = 1/2$	-3.62%	0.59%	0.23%
$\gamma = 3$	2.27%	18.50%	9.58%
$\gamma = 5$	34.63%	68.14%	47.49%
$\overline{H} = 100$	-2.88%	4.67%	1.46%
$\overline{H} = 150$	2.27%	18.50%	9.58%
$\overline{H} = 200$	14.51%	66.07%	34.59%
$\sigma_\omega = 0.05$	-0.87%	10.96%	5.31%
$\sigma_\omega = 0.20$	2.27%	18.50%	9.58%
$\sigma_\omega = 0.35$	15.47%	46.75%	26.31%

Conclusion

- The problem of mortgage choice is both basic and complex.
  - Basic because almost every middle-class household faces this choice at least once in his or her life.
  - Complex because it involves many considerations that are at the frontier of finance theory:
    - uncertainty in inflation and real interest rates
    - borrowing constraints
    - illiquid assets
    - uninsurable risk in labor income
    - the need to plan over a long horizon.
- In this paper we have shown that the form of the mortgage contract can have large effects on household welfare.

- The exact levels of welfare depend on the particular premia we have assumed for ARM and FRM mortgages, but we can draw general conclusions about the types of households that should be more likely to use ARMs. Households with:

  - smaller houses relative to income
  - more stable income
  - lower risk aversion
  - a higher probability of moving

  should be the households that find ARMs most attractive.

- These results match quite well with empirical evidence reported by Shilling, Dhillon, and Sirmans (1987).

  - Households with co-borrowers and married couples (whose household income is presumably more stable), and households with a higher moving probability, are more likely to use ARMs.

- Inflation-indexed mortgages offer large welfare gains given the inflation process we estimated for 1962–1999.