

# Shape Optimization for Trailing Edge Noise Control

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*with*

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*Supported by ONR*



# Motivation

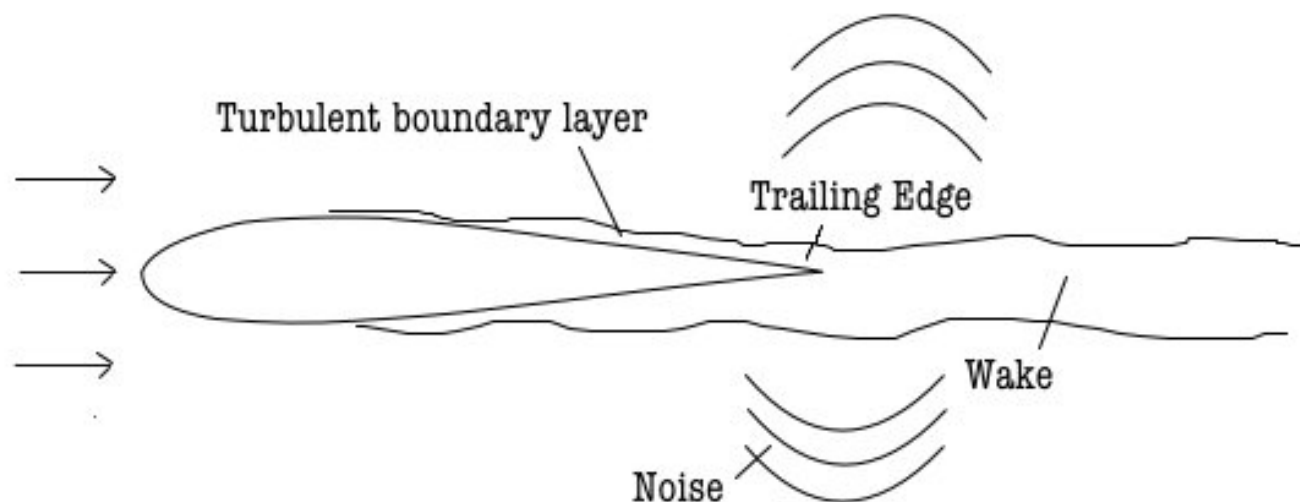
## ■ *Noise generated by flow past trailing edge of lifting surfaces*

□ laminar flow

- single tone from vortex shedding

□ turbulent flow

- tonal and broadband noise



# Motivation

## ■ *Why Reduce Noise?*

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## ■ *Development of design methods for “real life” fluids problems*

- ☐ unsteadiness
- ☐ turbulence

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- ☐ aircraft and airframe noise
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## ■ *Development of design methods for “real life” fluids problems*

- ☐ unsteadiness
- ☐ turbulence

## ■ *Influence of surface shape on noise production*

# Objectives

- *To develop a feasible method of shape optimization for trailing edge noise control*
  - computationally affordable

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- *Interface with existing Navier stokes code*

- ☐ demonstrate design of quiet trailing edge

- ☐ 2-D unsteady laminar flow past small airfoil

- ☐ Fully turbulent trailing edge flow

# Noise Computation Methods

## ■ *Direct Numerical Simulation*

- Not suitable for low Mach number flows
- Acoustic wavelength  $\gg$  flow scales
  - huge computational domain
- acoustic energy  $\ll$  flow energy
  - need accurate numerics

# Noise Computation Methods

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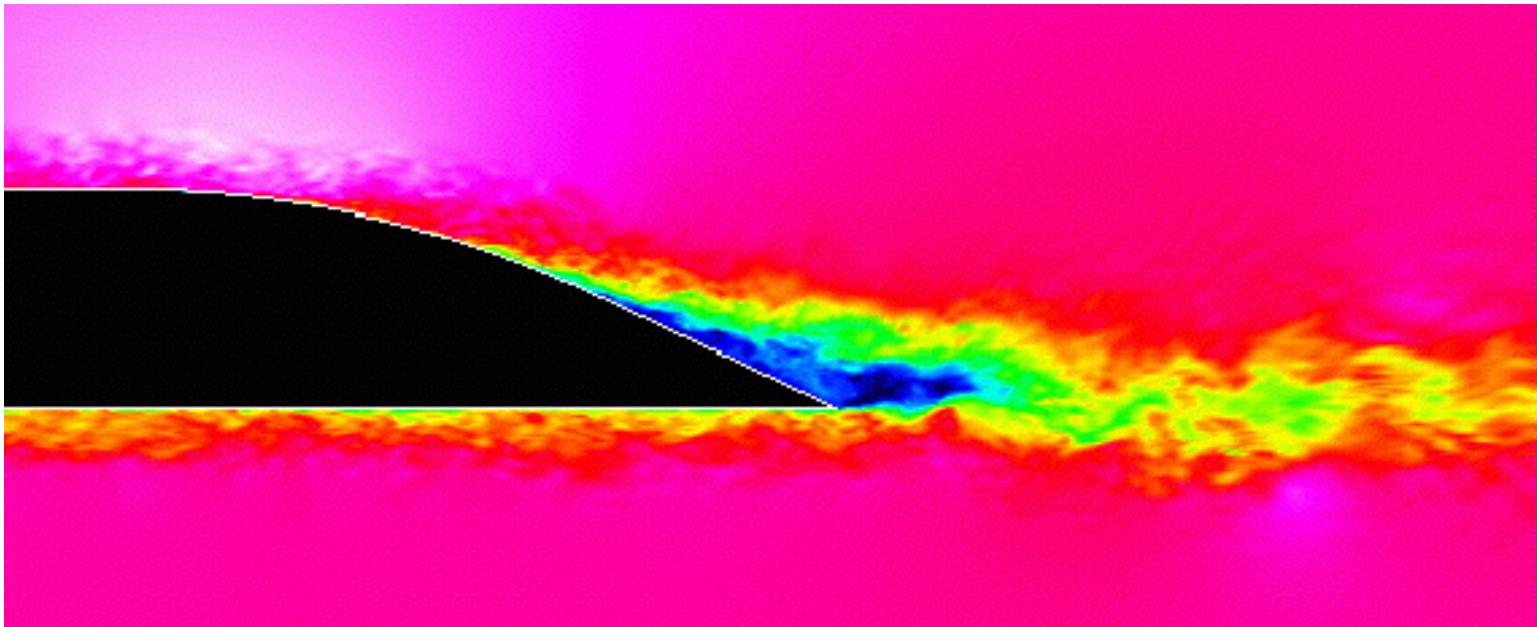
## ■ *Hybrid methods*

- good for small Mach number
- Lighthill Analogy
  - incompressible flow calculation  $\rightarrow$  source term for wave equation
  - takes advantage of scale separation

# Acoustic Computations

## ■ *Accurate simulation of trailing edge flow (Wang & Moin 2000)*

- Large eddy simulation
- $Re = 2.15 \times 10^6$ ,  $M = 0.09$
- Comparison with experiments of Blake (1975)



# Acoustic Computations

## ■ *Full Navier-Stokes solver*

- ☐ Sub-grid scale turbulence model
- ☐ Curvilinear coordinates

## ■ *Expense estimate for one flow solution*

- ☐ Converged turbulence and noise statistics
- ☐ 7 million mesh points
- ☐ 10,000 SGI single processor hours
- ☐ approximately 2 weeks using 32 processors!

## ■ *Need optimization method with minimum function evaluations!*

# Shape optimization for fluids

## ■ *Gradient based methods*

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## ■ *Gradient based methods*

### □ Direct

- Compute gradients by “brute force”
- Cost per iteration is  $\approx$   
# parameters  $\times$  simulation cost
- Expensive for many parameters
- Not widely used

# Shape optimization for fluids

## ■ *Gradient based methods*

### □ Direct

- Compute gradients by “brute force”
- Cost per iteration is  $\approx$   
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### □ Adjoint

- Compute gradients by solving adjoint equation
- Cost per iteration is  $\approx 2 \times$  simulation cost
- Need new adjoint solver for each flow solver
- Data storage issues for unsteady flow
- Used for aerodynamic design - steady flows

# Shape optimization for fluids

- Incomplete Sensitivities (Mohammadi and Pironneau 2000)
  - New method
  - Demonstrated for simple cases
  - Compute gradients by approximation
  - Cost per iteration is  $\approx$  simulation cost
  - Portable and inexpensive

# Shape optimization for fluids

## ■ *Non-gradient based methods*

### □ Evolutionary Algorithms

- Good for noisy cost functions
- Numerical simulations always have some noise
- Expense depends on number of parameters

### □ EA with response surface method

- Construct approximation of cost function during iterations
- Reduced cost compared to other EA's

# Gradient Evaluation

- *In general, cost function depends on geometry and state*

$$\frac{dJ}{da_i} = \frac{\partial J}{\partial a_i} + \frac{\partial J}{\partial q_j} \frac{\partial q_j}{\partial a_i} + \frac{\partial J}{\partial U_k} \frac{\partial U_k}{\partial q_j} \frac{\partial q_j}{\partial a_i}$$

$a_i = \text{control variables}$

$q_i = \text{geometric variables}$

$U_i = \text{state (flow) variables}$

- Classic methods are expensive, especially for unsteady flows
  - Direct computation
  - Adjoint method

# Incomplete Sensitivities

- *Mohammadi and Pironneau (2000)* suggest that if  $J$  is defined on a surface

$$\frac{dJ}{da_i} \approx \frac{\partial J}{\partial a_i} + \frac{\partial J}{\partial q_j} \frac{\partial q_j}{\partial a_i}$$

- Sensitivity to state negligible relative to geometric sensitivity
- No need to solve adjoint problem
- Independent of flow solver
- Computational cost  $\approx$  simulation cost

# Optimization Procedure

- Parameterize surface deformation (polynomials or splines)

$$\delta y = \sum_{i=1}^n a_i x^i, \quad a_i = \text{control parameters}$$

- Advance flow solver in time until statistically steady
- Evaluate the gradients until converged

$$\frac{dJ}{da_i} = \frac{J(a_i + \epsilon) - J(a_i)}{\epsilon}, \quad \text{and} \quad \overline{\frac{dJ}{da_i}} = \frac{1}{T} \int_0^T \frac{dJ}{da_i} dt$$

- Calculate shape deformation with steepest descents

$$\delta a_i = -\lambda \overline{\frac{dJ}{da_i}}, \quad \delta y = \sum_{i=1}^n (a_i + \delta a_i) x^i$$

- Modify shape and generate new grid, repeat

# Cost Function Definition

## ■ *Model Problem: Unsteady, laminar flow*

□ 2-D Acoustically compact airfoil

- Noise from unit span given by Curle's formula

$$\rho \approx \frac{M^3}{4\pi} \frac{x_i}{|\vec{x}|^2} \dot{D}_i(t - Mr)$$

$$\dot{D}_i = \frac{\partial}{\partial t} \int_s n_j p_{ij}(\vec{y}, t - Mr) d^2 \vec{y}, \quad p_{ij} = p \delta_{ij} - \tau_{ij}$$

- Total acoustic power  $\propto \overline{\dot{D}_1^2} + \overline{\dot{D}_2^2}$

□ Cost function is defined as

$$J = \overline{\left( \frac{\partial}{\partial t} \int_s n_x p(\vec{y}, t) dS \right)^2} + \overline{\left( \frac{\partial}{\partial t} \int_s n_y p(\vec{y}, t) dS \right)^2}$$

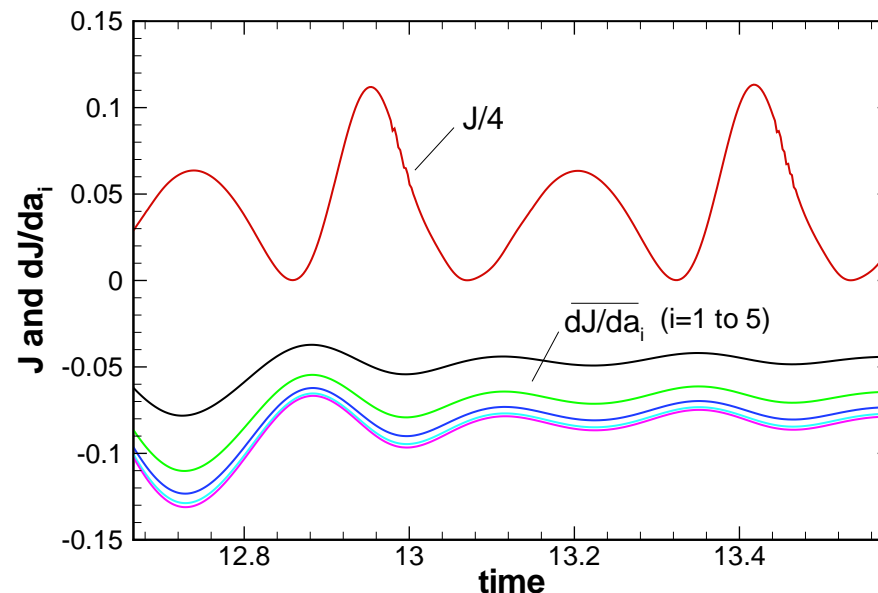
# Model Problem: Setup

□ Laminar flow over airfoil



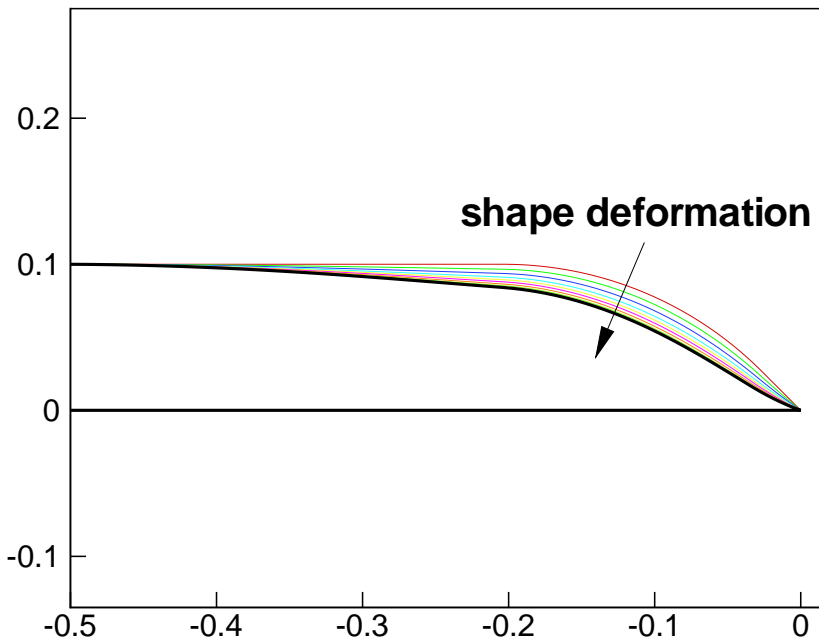
- Initial T.E. tip angle 45 degrees
- Allow section of upper surface to deform (blue section)

Time history of  $J$  and  $dJ/da_i$  ( $Re=10,000$ , initial shape)

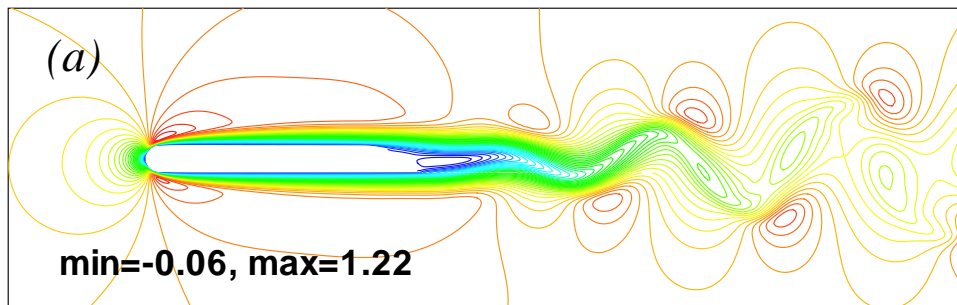
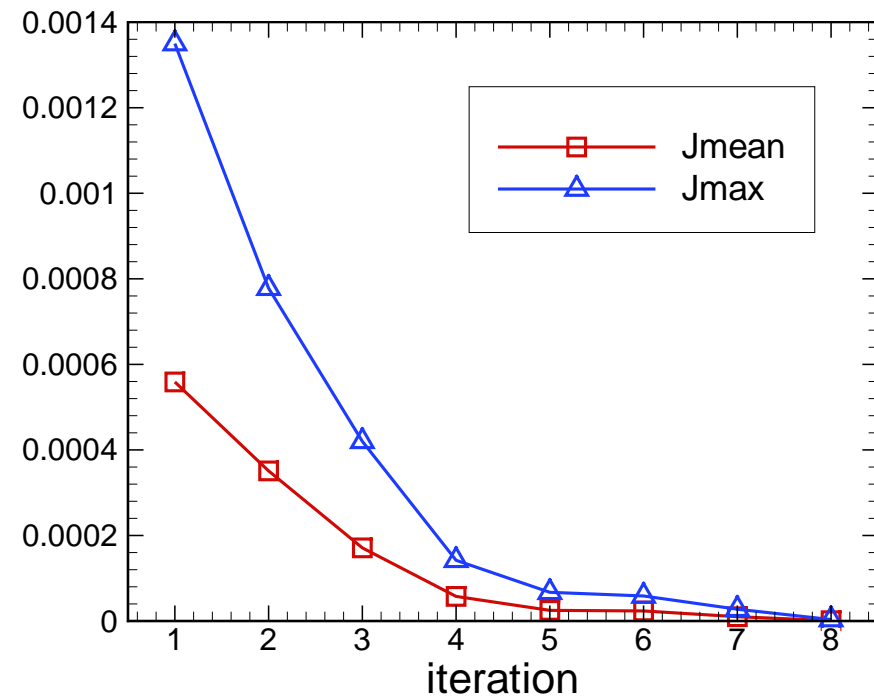


# Model Problem: $Re = 2,000$

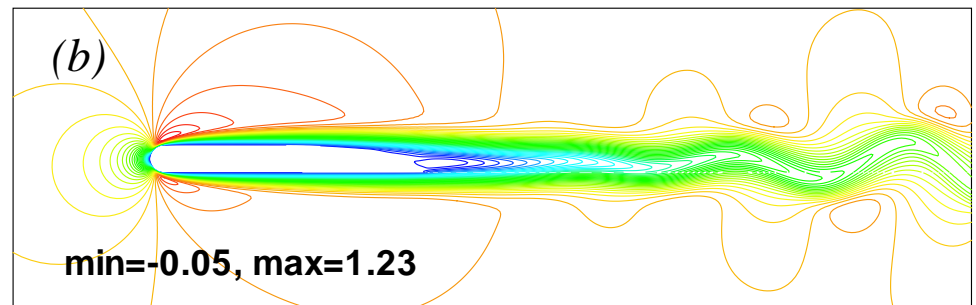
## Shape Evolution



## Cost Function Evolution



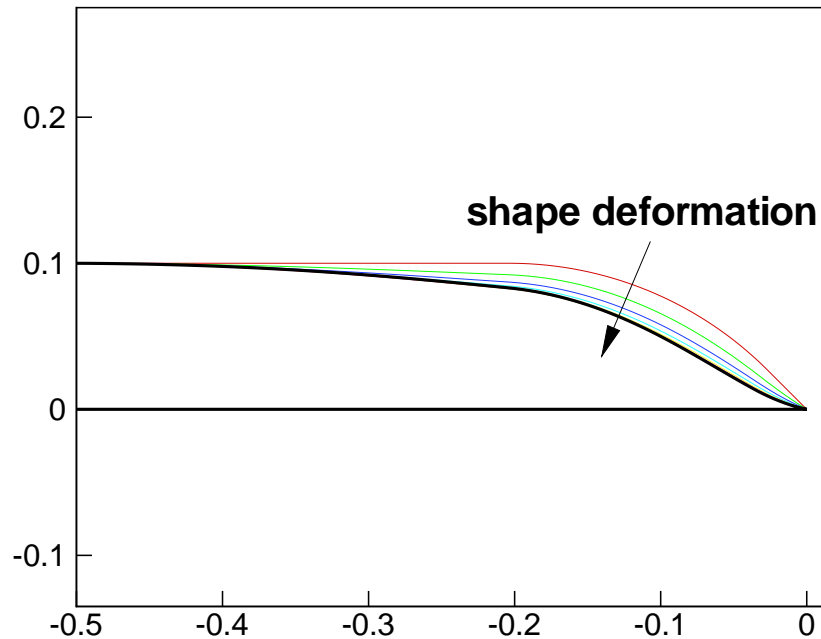
Initial velocity contours



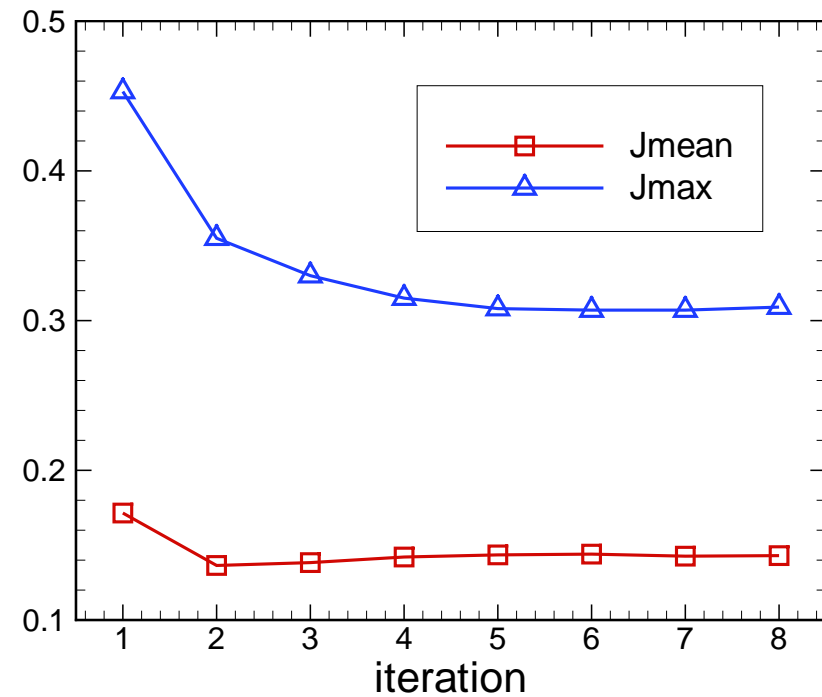
Final velocity contours

# Model Problem: $Re = 10,000$

## Shape Evolution

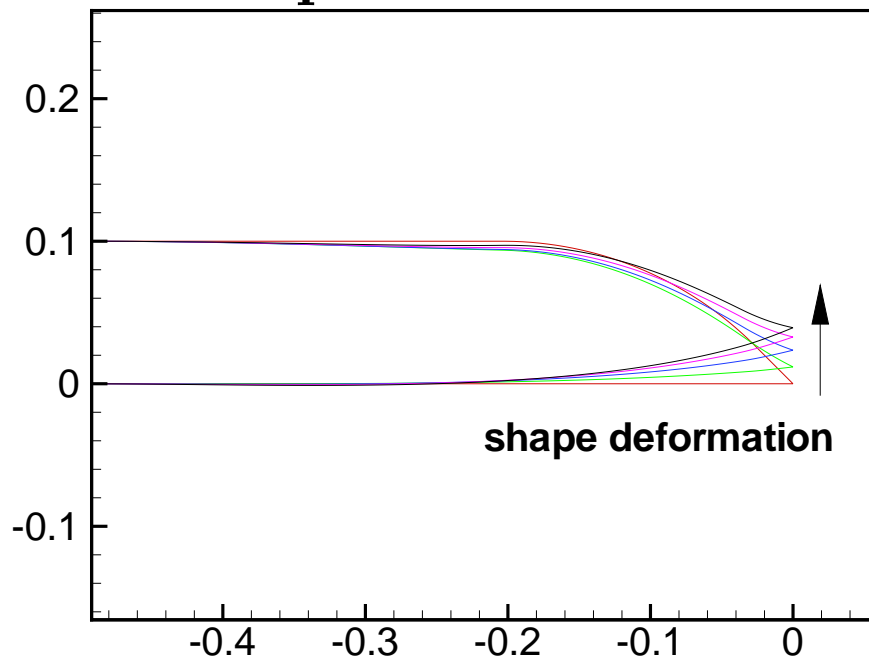


## Cost Function Evolution

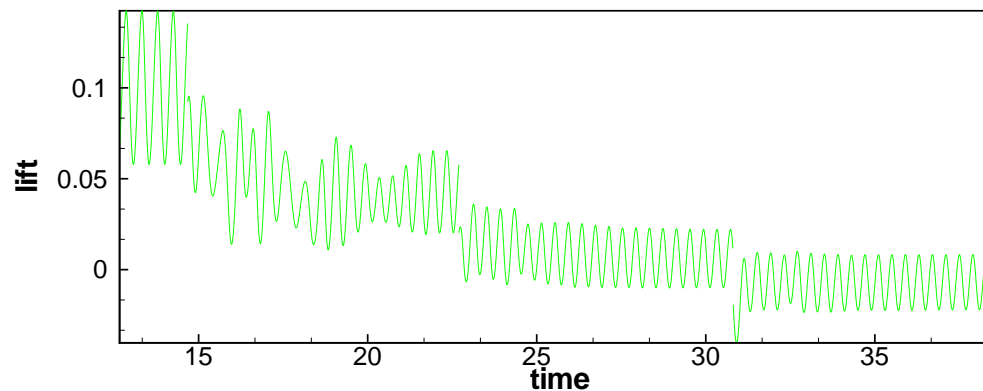
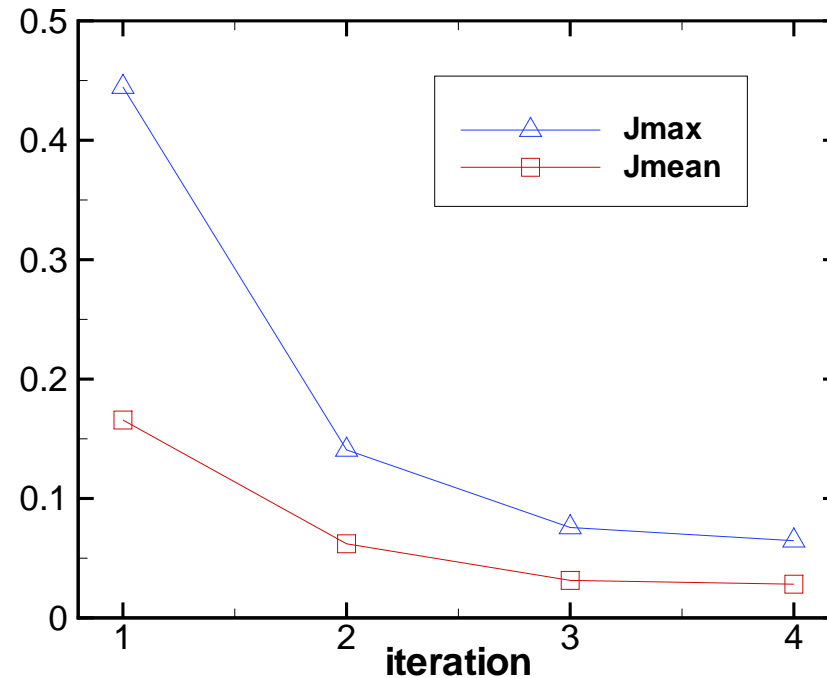


# Both Sides: $Re = 10,000$

## Shape Evolution



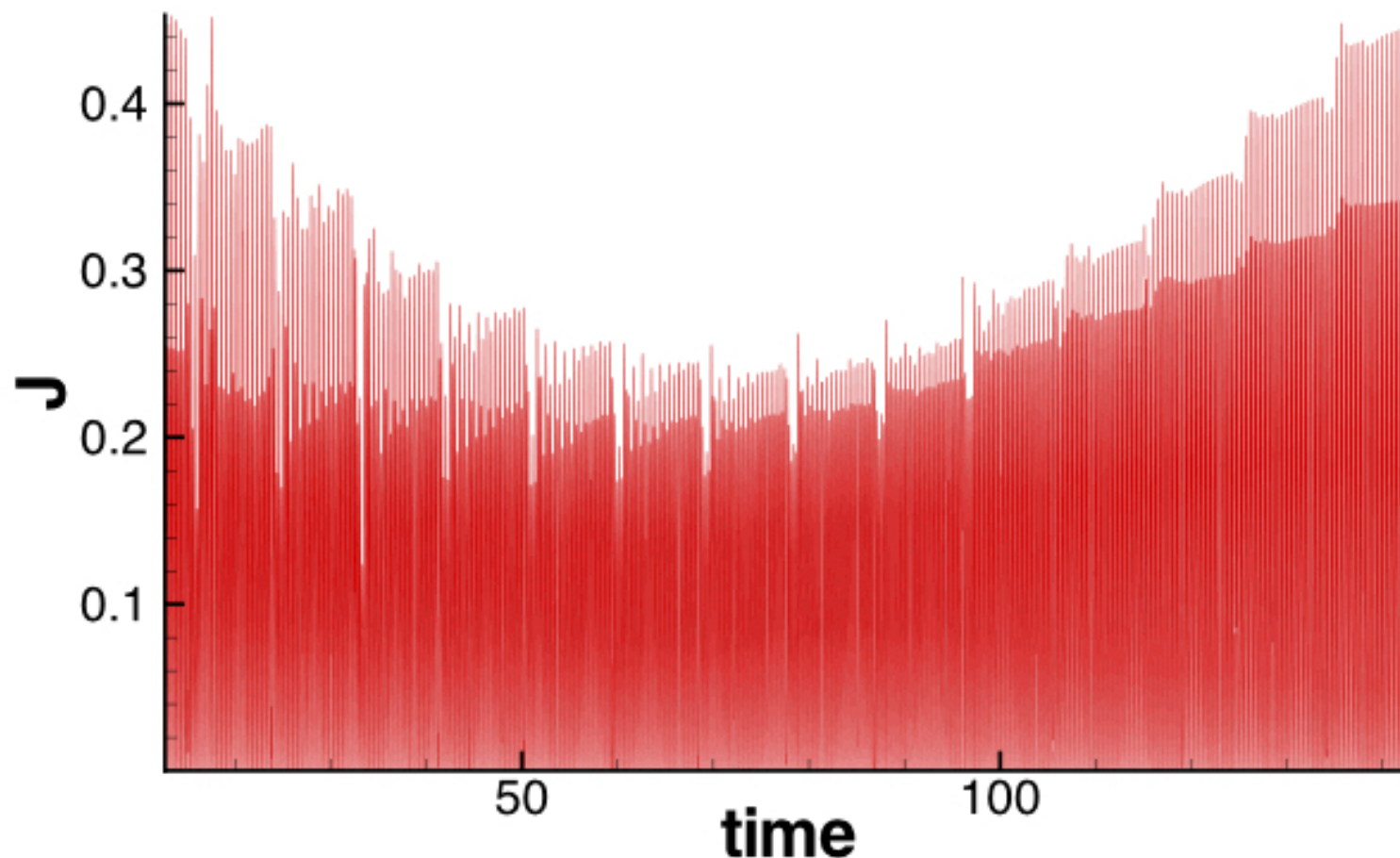
## Cost Function Evolution



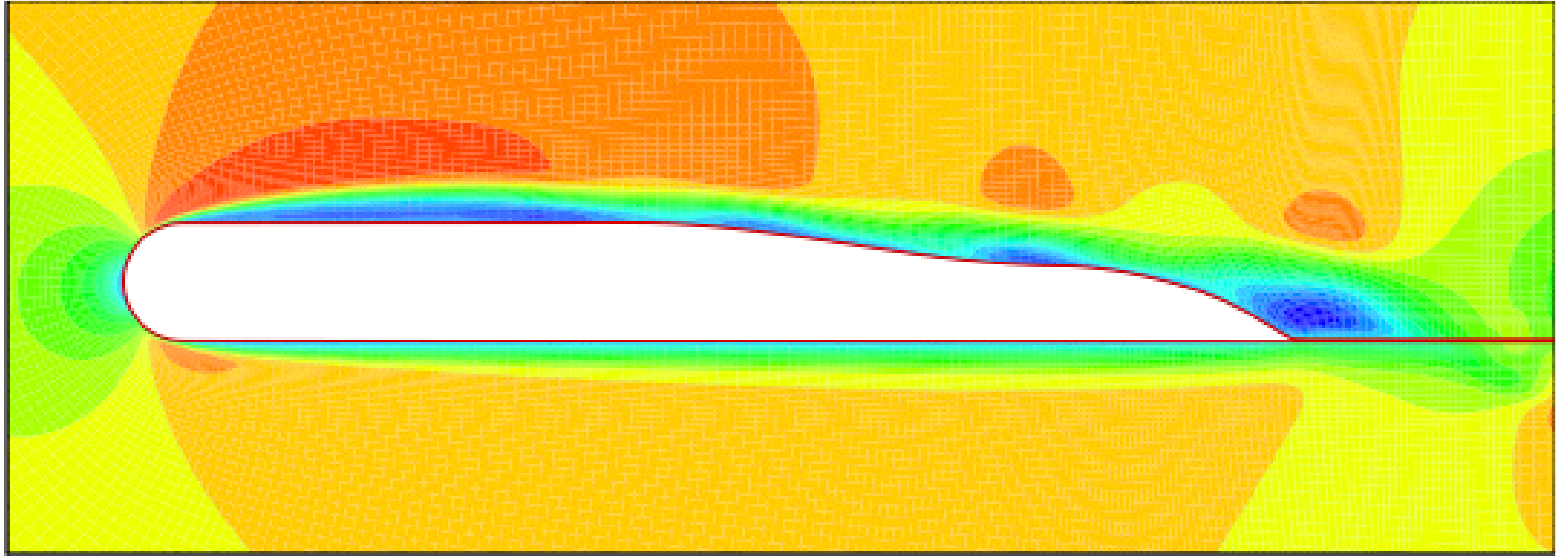
## Change in Lift

# Assumption Breaks Down

- *Continue to iterate  $\rightarrow$  shape does not converge  $\rightarrow$  cost function increase!*



# Assumption Breaks Down

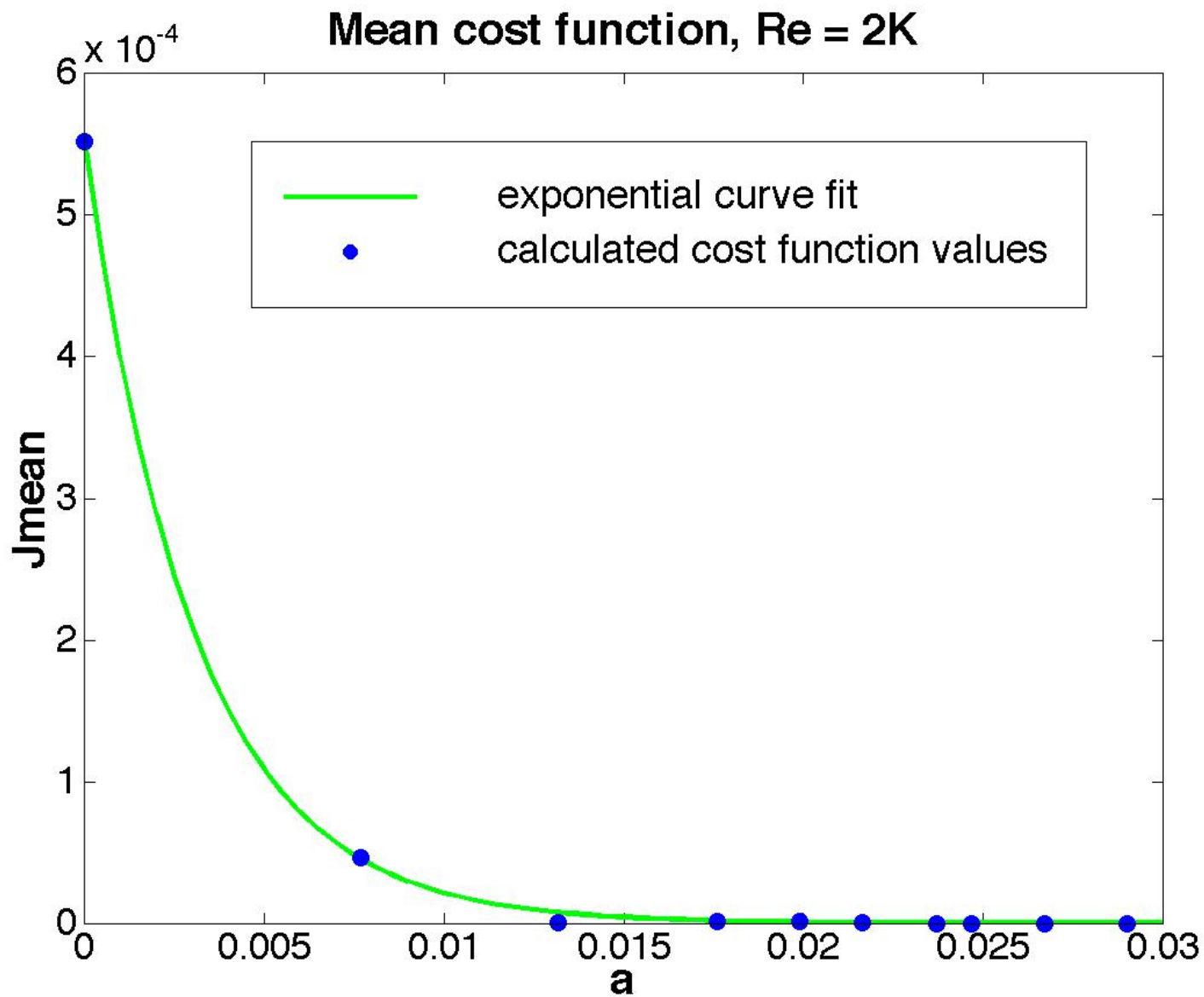


- *Cost function increases when flow separates*
- *Gradient is wrong*
  - Incomplete assumption invalid ?

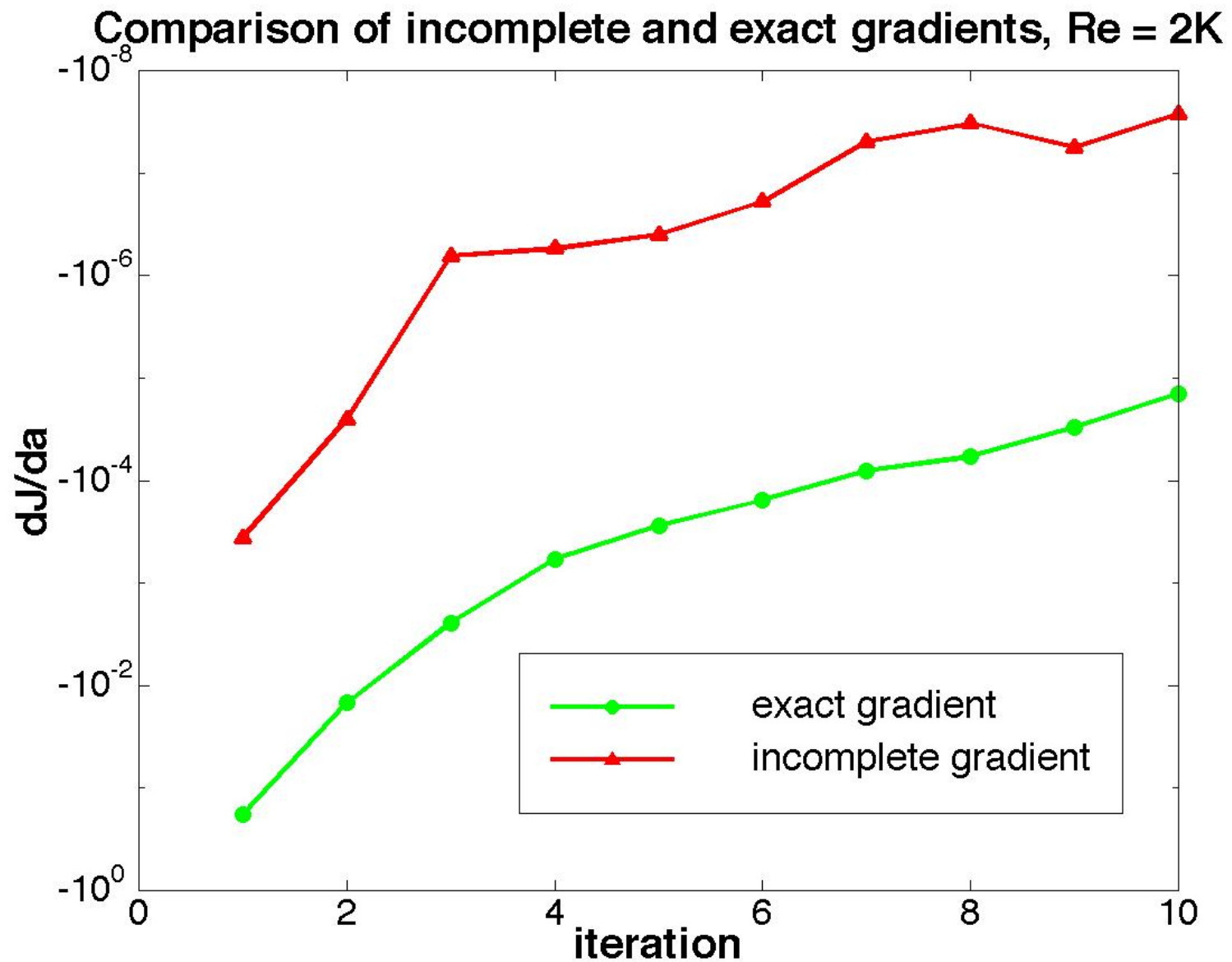
# Compute Exact Gradients

- *Test validity of incomplete assumption*
- *With one spline case, compute exact gradients by “brute force”*
  - Find exact gradient with finite difference
  - Convergence for exact gradient is difficult
    - *integration of oscillatory cost function*
  - Do curve fit of cost function vs. displacement
    - *take derivatives of fit*
  - Compare exact and incomplete gradients

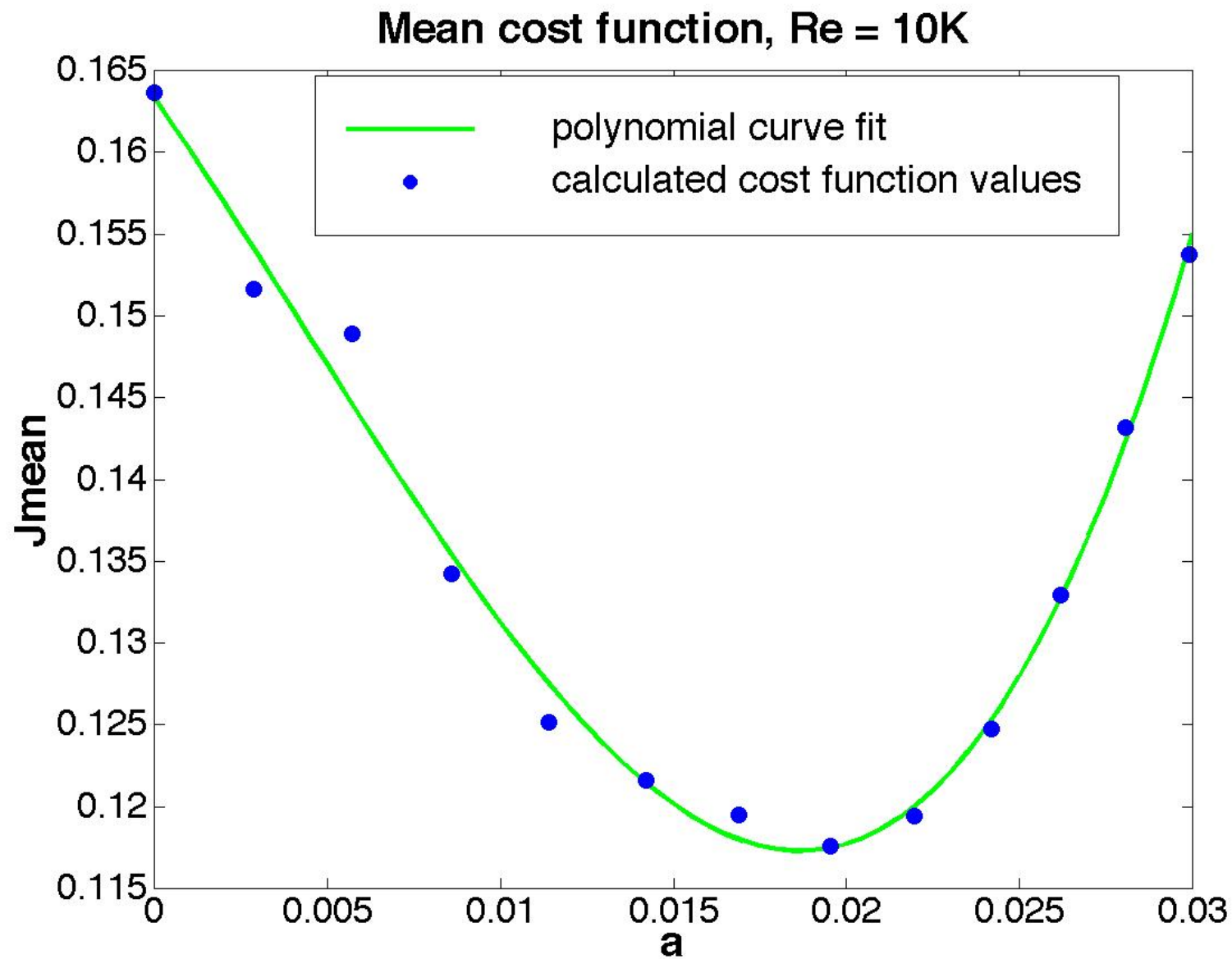
# Compute Exact Gradients



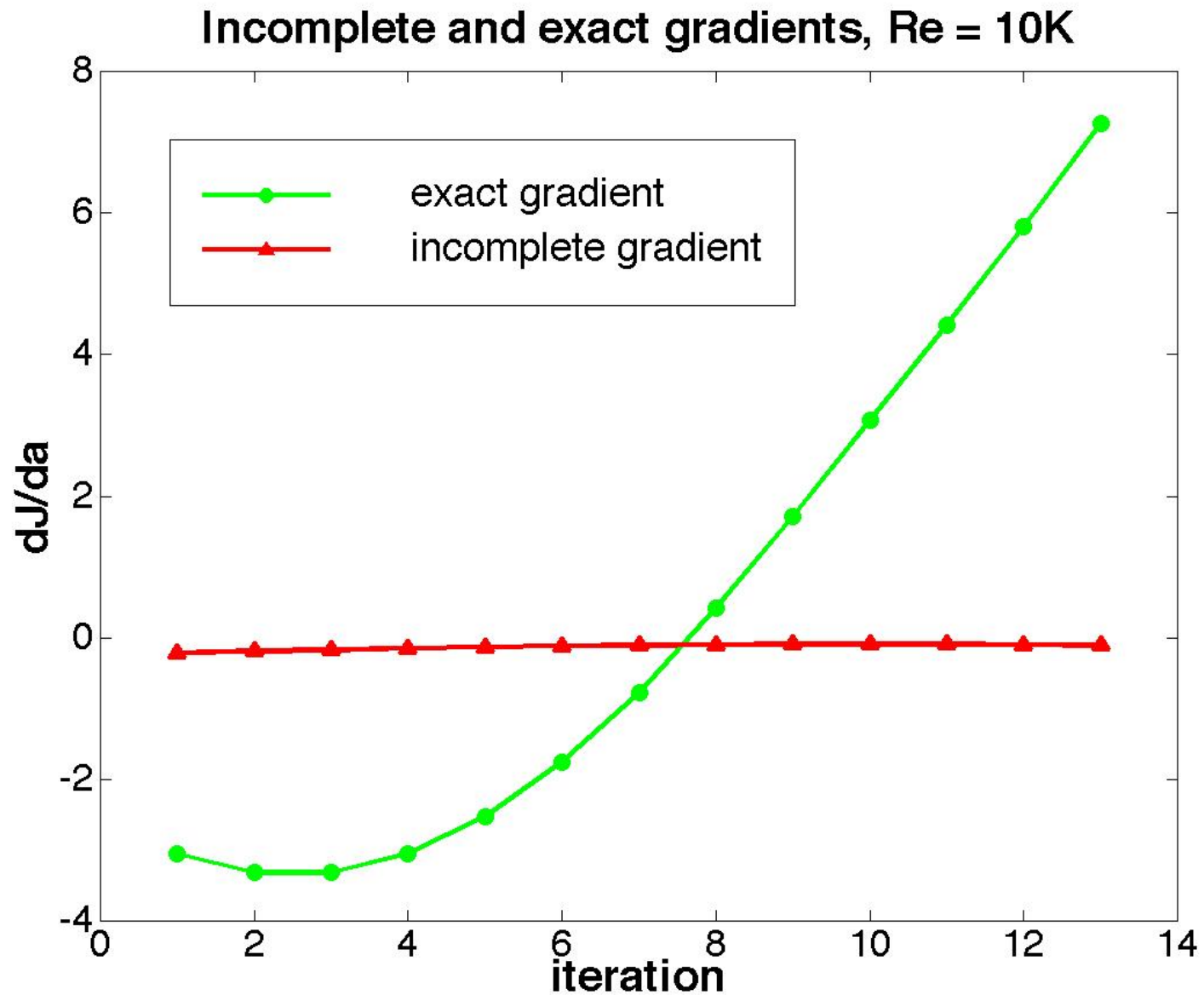
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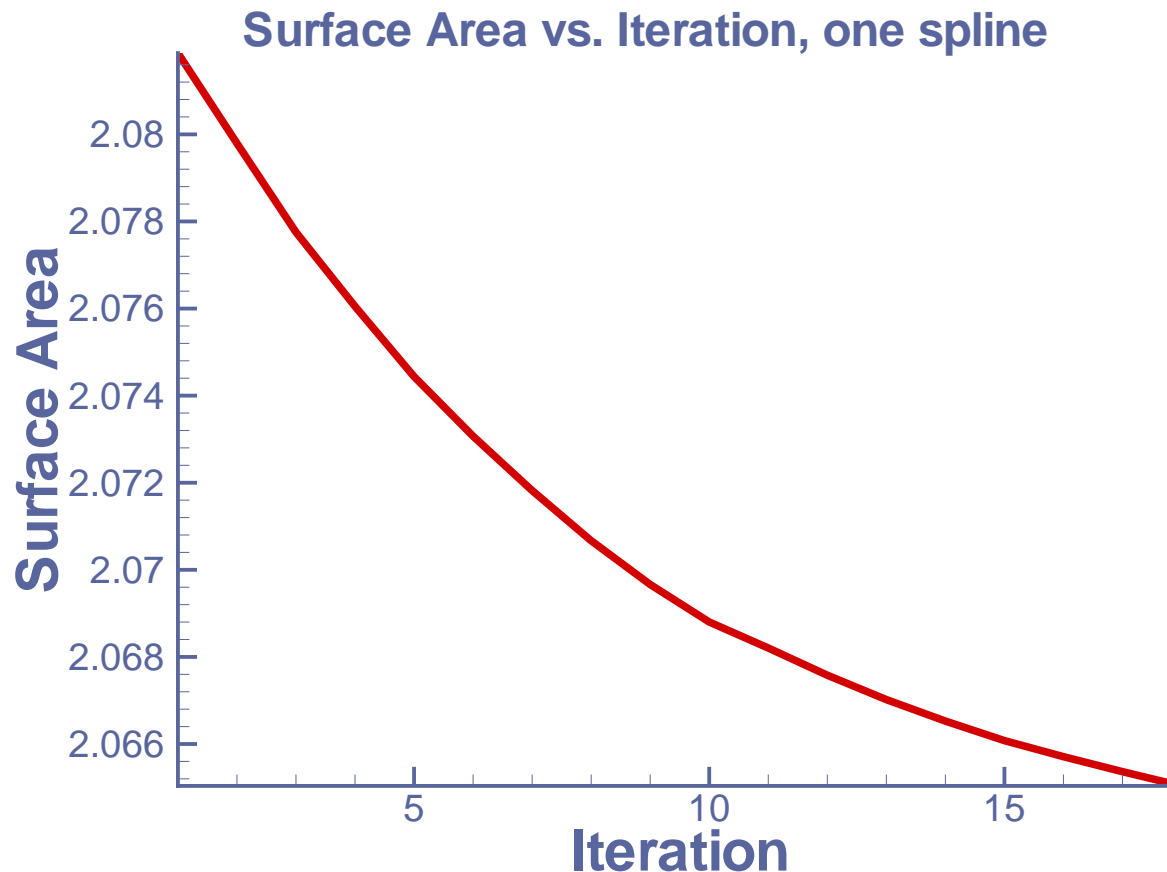


# Compute Exact Gradients



# What is minimized?

- *Incomplete assumption only accounts for geometry at each time step*
- *Method minimizes surface area?*



# Conclusions and Future Work

- *No free lunch!*
- *Incomplete sensitivities assumption not valid for this case*
- *Choose new method*
  - Adjoint
    - optimal control
    - suboptimal control
  - Evolutionary algorithm
    - Traditional method
    - Surface response method
  - Comparison of both?

# Conclusions and Future Work

- *Extension to high  $Re$  turbulent trailing edge flow*
  - Cost function identification: define on surface
  - Total radiated power vs. frequency-weighted power
    - Low frequency noise propagates further
- *Add constraints: lift, drag, thickness, volume, etc.*