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Stochastic Exposures & LGDs in Portfolio Credit Risk, and their impact in BIS II capital

FIELDS Institute

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Know *Your* Risk

Case Study: Key Points

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- Standard treatment of portfolio credit risk assumes deterministic exposures (LGDs)
  - If stochastic, they are generally assumed independent
- We use a stylized case study to demonstrates the impact of stochastic & correlated exposures on credit capital (results generally extend to LGDs & collateral)
  - Substantial effect on losses from vols., credit & market-credit correlations, granularity
- Results have important implications for both economic and regulatory Capital
- While analytical approximations might be possible sometimes, substantial benefits from
  - Accurate models for exposures, LGDs, collateral and other mitigation techniques
  - A flexible, integrated market and credit-simulation model

Outline

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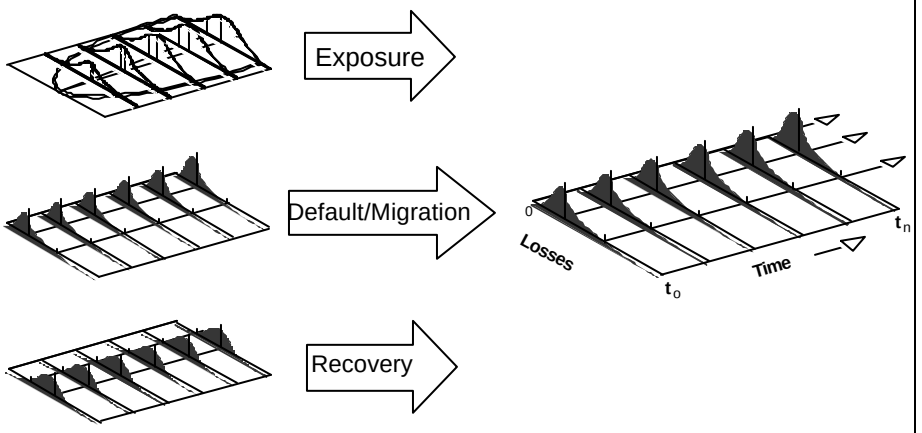
- Introduction
  - Credit capital and BIS II
  - Credit Exposures for derivatives
- General Portfolio Credit Risk Framework
  - Integrated Market-Credit Risk
  - Portfolio Model in BIS II weights
- Case study
  - Credit Risk with Stochastic Exposures/LGDs

Elements of Credit risk



Outline

- Introduction
  - Credit capital and BIS II

BIS II Proposal for new capital adequacy framework

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Three pillars:

- Minimum capital requirements
  - Gives the explicit rules that define the minimum ratio of capital to risk weighted assets
- Supervisory review process
  - Requires supervisors to undertake a qualitative assessment of capital allocation techniques and compliance with standards actually in place in an institution
- Market discipline
  - High disclosure standards & adequate capital which facilitate market discipline

Minimum Capital Under BIS II

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Summary of minimum capital requirements

- Three approaches to calculation of risk-weighted assets:
  - (Revised) standardized approach
  - Foundation internal ratings-based (IRB) approach
  - Advanced Internal ratings-based (IRB) approach
- Explicit capital charge for operational risk
- Market risk capital as defined in the 1996 Amendment to remain largely unchanged

BIS II Advanced IRB Approach

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$$\text{Regulatory Capital} = \left(\sum_j^n E_j \cdot RW_j \right) \times 8\% + GA$$

GA = *Granularity Adjustment*
RW_j = Risk Weight for asset/obligor j
E_j = Exposure at default for asset/obligor j

- RW x E x 8% represents the capital for a "perfectly" diversified portfolio (asymptotically fined grained; with only systemic risk)
- The "granularity adjustment" adjusts the capital for the level of diversification of the actual portfolio
  - In previous versions of proposal but may not be part of the final framework

BIS II Advanced IRB Approach

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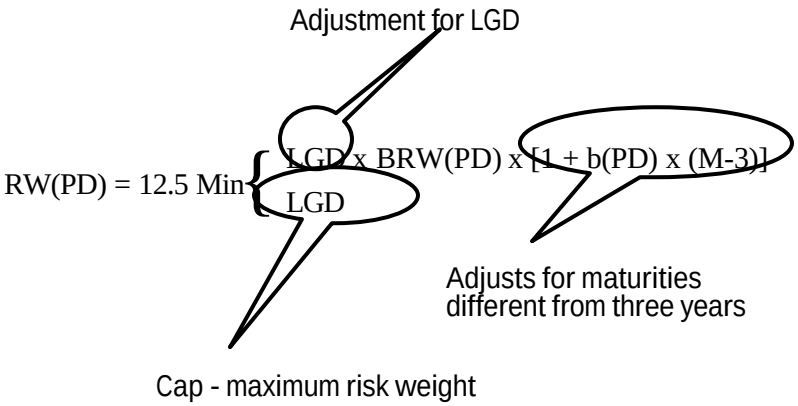

BIS II Advanced IRB Approach

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Expected and unexpected losses of a hypothetical, asymptotically fine-grained, portfolio of one-year loans (from one-factor credit portfolio model)

$$BRW(PD) = N\left[(1-R)^{-0.5} N^{-1}(PD) + 3.09 \cdot (R/(1-R))^{0.5}\right] [1 + 0.0470 \cdot (1-PD)/PD^{0.44}]$$

where
R= Correlation
(inverse relationship with PD)

Maturity adjustment to reflect a portfolio of maturity 3 years

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  - Credit Exposures for derivatives

Evolution of Credit Exposure Measurement & Limits

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First level (common practice): Exposure = Notional

- Simple, easy
- Generally used for traditional loan products
- handles derivatives as loans
- Does not consider "potential exposures"

Second level (BIS I): Exposure = MtM + Add On (potential exposure)

- Easy to implement
- Better for derivatives
- May not capture properly offsets, netting, mitigation

Third level: Exposure profile over time - Simulation

- Accurate for derivatives... but computationally intensive
- Multiple time limits

Derivatives Exposures in Standard Model (Capital Accord '88)

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Total Exposure = Actual Exposure + Potential Exposure

- Actual Exposure = Mark to Market
- Potential Exposure = Notional X *counterparty add-on*
  - *add-on= F* (instrument, maturity, currency)

Exposure Simulation (multi-step)

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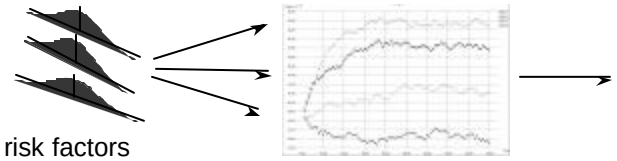
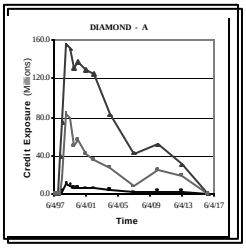
- Multi-step Scenarios: model joint process of risk factors affecting prices (IRs, FX, EQ,…)
- Value instruments and sum (net) - each time & scenario
- (Dynamically) apply collateral and mitigation
- Aggregate to create exposure distribution



risk factors

scenarios



Advanced Exposure Modelling

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- Credit Mitigation
  - Netting
  - Collateral, Margin calls
  - Credit triggers
- Credit Derivatives

CP\Underlying	Default	No default
Default		
No Default		

- Wrong Way Exposures: interdependency of the counterparty quality and the market factors (e.g. hedge funds, sovereign debt in foreign currency) -

Wrong Way Exposures

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Scenarios where transactions are in the money to us… likely to coincide with counterparty having difficulty fulfilling its obligation

Examples:

- Currency Swap with emerging market sovereign
  - sovereign pays USD: significant correlation: exposure and FX
- IR swap with highly leveraged institution which receives fixed
  - rising IRs will more likely result in defaults
- Some general empirical evidence: 1971-92 defaults tended to cluster in periods with falling IRs (Duffee 1996)
  - receiver swap: significant correlation between exposure and IRs
  - "correlated exposures" ~ 65% grater than measures assuming independence

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Portfolio Credit Risk Reports

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	Base Case
Expected Losses	95
Standard Deviation	232
Maximum loss (99%)	1,026
Credit VaR 99%	931
Maximum loss(99.9%)	1,782
Credit VaR 99.9%	1,687
Shortfall 99%	1,320
Shortfall 99.9%	1,998

Statistics for one year loss distribution

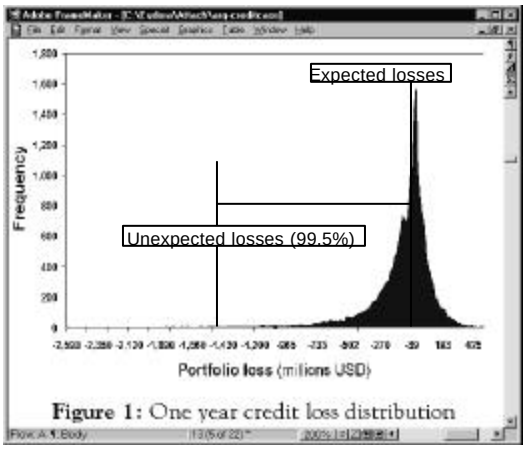

Figure 1: One year credit loss distribution

Mark-to-Future Framework for Credit Risk

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1. Risk factor Scenarios ("states of the world")
   - evolution of systemic (market & credit) risk factors over horizon
2. Joint default/migration model
   - economic conditions ---> default/migration
   - default/migration probabilities are conditioned on the scenario (correlations: joint variation of obligors probabilities over scenarios)
3. Obligor exposures, recoveries and losses in a scenario
   - Mark-to-Future exposures in a scenario (with netting, collateral, etc.)
4. Conditional portfolio loss distribution in a scenario
   - efficient computation: credit events of each obligor are independent
5. Aggregation of losses in all scenarios
   - average over all scenarios of conditional loss distributions

Mark-to-Future Framework for Credit Risk

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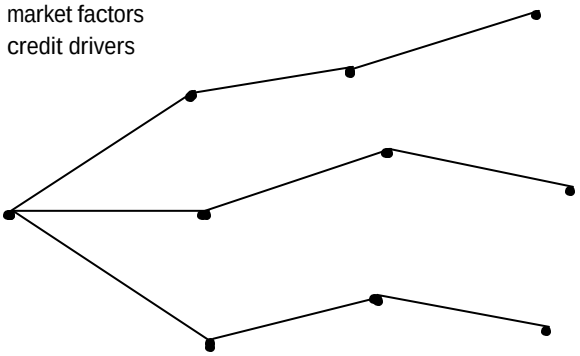
1. Scenarios:
market factors
credit drivers



X *(0)* ***X*** *(t1)* ***X*** *(t2)* ***X*** *(t3)*

Mark-to-Future Framework for Credit Risk

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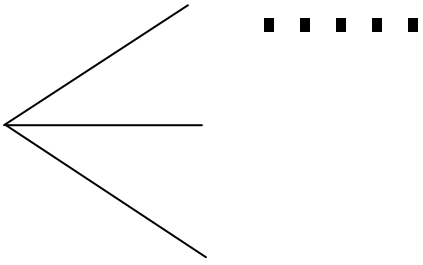
1. Scenarios:
market factors
credit drivers

2. Conditional obligor
default/migration
probabilities



Joint Default/Migration Models

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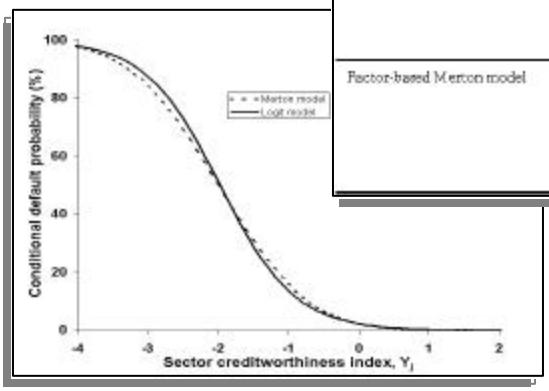
Model	Conditional default probabilities
Sector-based logit model	$p_j(Z^S) = \frac{1}{1 + a_j \exp\left\{b_j\left(\sum_k \beta_{jk}^{S*} Z_k^S\right)\right\}}$
Factor-based logit model	$p_j(Z) = \frac{1}{1 + a_j \exp\left\{b_j\left(\sum_k \beta_{jk}^{M*} Z_k^M + \beta_{jj}^{S*} Z_j^S\right)\right\}}$
Factor-based Merton model	$p_j(Z) = \Phi\left(\frac{a_j - \left(\sum_k \beta_{jk}^M Z_k^M + \beta_{jj}^S Z_j^S\right)}{\sigma_j}\right)$



Several Joint Default/Migration Models

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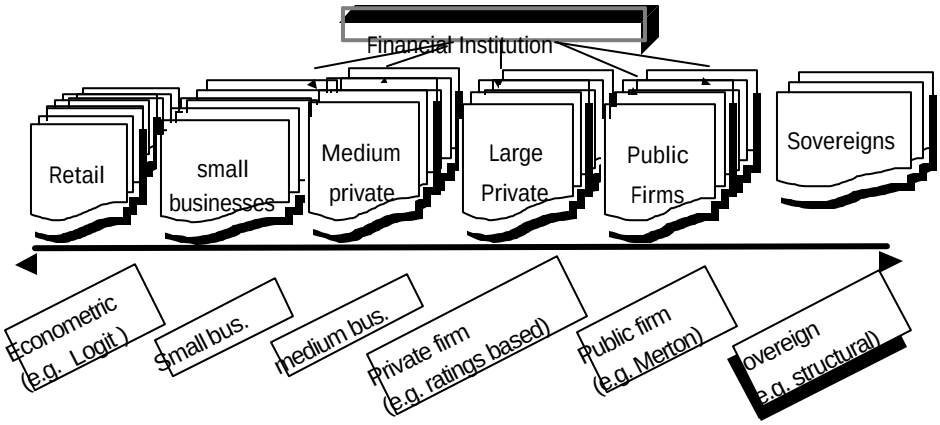

Mark-to-Future Framework for Credit Risk

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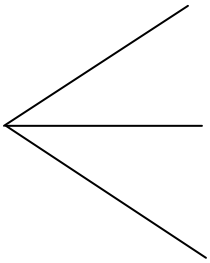
1. Scenarios: market factors credit drivers

2. Conditional obligor default probabilities

3. Obligor scenario losses (exposures X LGD)



Counterparty Exposures through Mark-to-Future

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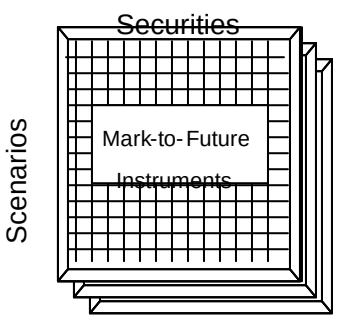
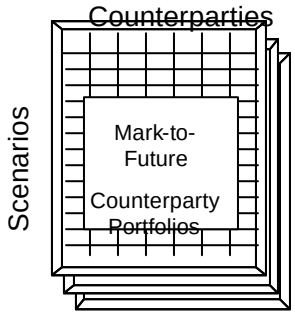
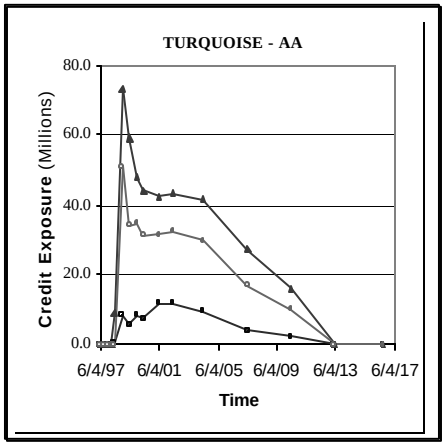
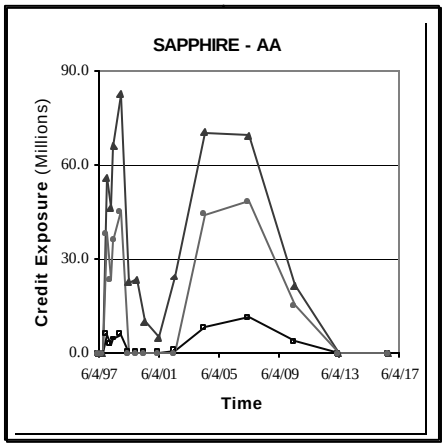
aggregation, netting, collateral, credit mitigation, etc.





Exposure Profiles & Limits





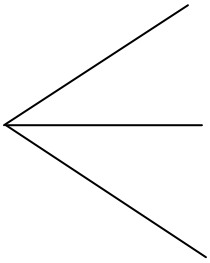
Mark-to-Future Framework for Credit Risk

1. Scenarios:
   - market factors
   - credit drivers

2. Conditional obligor default probabilities

3. Obligor scenario losses (exposures X LGD)

4. Conditional portfolio losses



Mark-to-Future Framework for Credit Risk

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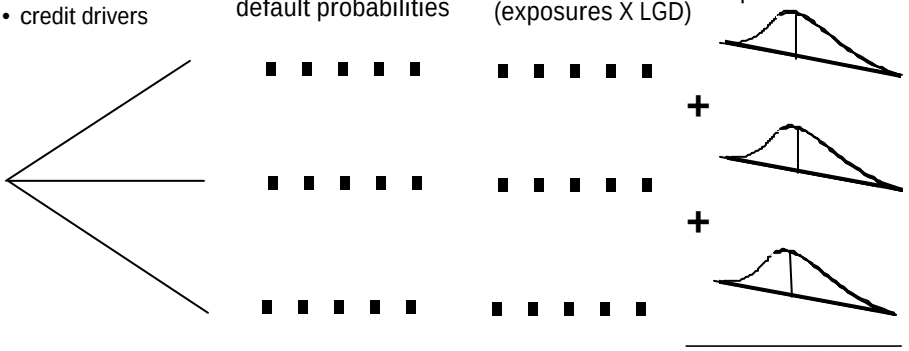
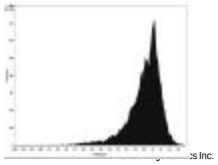


5. Unconditional Portfolio loss distribution



Systemic and Idiosyncratic Portfolio Losses

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Systemic losses are

- The actual losses of an asymptotically fined grained portfolio. Formally, it is
  - the portfolio we obtain by dividing every exposure into *n* identical exposures
  - taking the limit as *n* gets very large
- Conditional on a given scenario, the loss distribution collapses to a single point → its expected loss;
  - all higher moments vanish
- Consequence of the Law of Large Numbers (LLN) and the property of scenario conditional independence.
- It can be a good approximation for large, well diversified, portfolios

Systemic and idiosyncratic Portfolio Losses

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1. Scenarios
2. Conditional probabilities $\boldsymbol{p(X)}$
3. Obligor losses $l(\boldsymbol{X})$
4. Conditional (total) portfolio losses $P(L = l|\boldsymbol{X})$
4a. Conditional Systemic losses $\mathrm{E}\{L|\boldsymbol{X}\}$

$\boldsymbol{X}=\boldsymbol{x}_1$ $p_j(\boldsymbol{X}=\boldsymbol{x}_1)$ $j=1,\ldots,n$ $l_j(\boldsymbol{X}=\boldsymbol{x}_1)$ $j=1,\ldots,n$ $E\{L \mid X = x_1\} = \sum_{j=1}^{n} l_j \cdot p_j\{x_1\}$

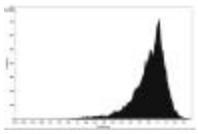
+

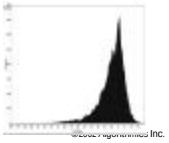
$\boldsymbol{X}=\boldsymbol{x}_2$ $p_j(\boldsymbol{X}=\boldsymbol{x}_2)$ $j=1,\ldots,n$ $l_j(\boldsymbol{X}=\boldsymbol{x}_2)$ $j=1,\ldots,n$ $E\{L \mid X = x_2\} = \sum_{j=1}^{n} l_j \cdot p_j\{x_2\}$

+

$\boldsymbol{X}=\boldsymbol{x}_3$ $p_j(\boldsymbol{X}=\boldsymbol{x}_3)$ $j=1,\ldots,n$ $l_j(\boldsymbol{X}=\boldsymbol{x}_3)$ $j=1,\ldots,n$ $E\{L \mid X = x_3\} = \sum_{j=1}^{n} l_j \cdot p_j\{x_3\}$

5. Unconditional Portfolio loss distribution





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Portfolio Credit Risk Modelling in BIS II

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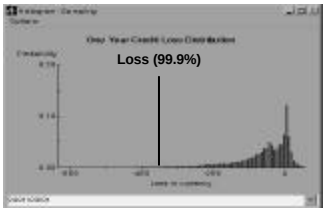
BIS II IRB approach embeds already a portfolio model

Modelling objective: to develop risk-bucketing capital rules consistent with a portfolio credit risk model:

- Weights must be additive
- Weights must be portfolio invariant



- Under what modelling assumptions does a portfolio model yield portfolio-invariant marginal risk contributions?

Portfolio Credit Risk Modelling in BIS II

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Modelling objective → Answer (Gordy) :

- Portfolio-invariant marginal risk contributions (additive and portfolio invariant weights) only if two conditions hold:
  - Asymptotically fined grained portfolio (systemic losses)
  - Single systemic risk factor
  - Independent EADs/LGDs
- Analysis of rates of convergence → simple approximation for "portfolio level" add-on charge for undiversified, idiosyncratic, risk
  - AFGP assumption is thus generally not a practical problem
- Single factor assumption is recognized to cause an important limitation

BIS II Advanced IRB Approach

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Expected and unexpected losses of a hypothetical, asymptotically fine-grained, portfolio of one-year loans (from one-factor credit portfolio model)

$$BRW(PD) = N\left[(1-R)^{-0.5} N^{-1}(PD) + 3.09 \cdot (R/(1-R))^{0.5}\right] [1 + 0.0470 \cdot (1-PD)/PD^{0.44}]$$

where
R= Correlation

Maturity adjustment to reflect a portfolio of maturity 3 years

BIS II Portfolio Model

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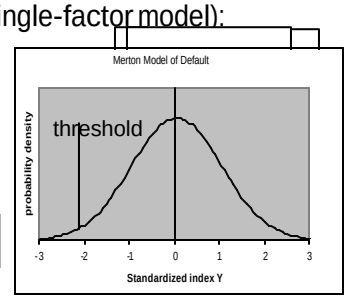
Creditworthiness model – 1-factor Merton

- Credit worthiness Index of obligor j (linear single-factor model):

$$Y_j = b\ x + \sqrt{1-b^2}\,e$$

- Conditional default probability

$$p_j(x) = p\left(p_j(x) = N\left(\frac{N^{-1}(p_j) - b\ x}{\sqrt{1-b^2}}\right) \mid x\right)$$



where p_j is the unconditional default probability (average of $p_j(x)$)

- note that the default threshold is $N^{-1}(p_j)$

BIS II Portfolio Model – systemic risk

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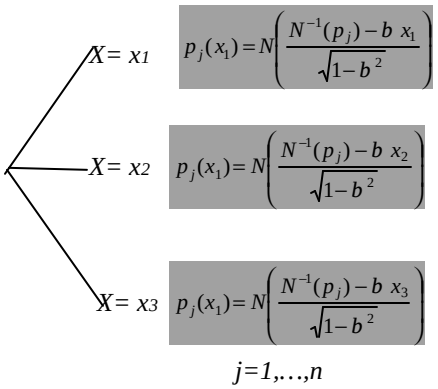
1. Scenarios
2. Conditional probs. $\boldsymbol{p}(X)$
3. Obligor (average) losses $l(X)$
4. Cond. systemic losses $\mathrm{E}\{L|\boldsymbol{X}\}$



$$p_j(x_1) = N\left(\frac{N^{-1}(p_j) - b\ x_1}{\sqrt{1-b^2}}\right)$$

$$p_j(x_1) = N\left(\frac{N^{-1}(p_j) - b\ x_2}{\sqrt{1-b^2}}\right)$$

$$p_j(x_1) = N\left(\frac{N^{-1}(p_j) - b\ x_3}{\sqrt{1-b^2}}\right)$$

$j=1,\ldots,n$

BIS II Portfolio Model – systemic risk

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Under the 1-factor model, the q-th percentile systemic (IFGP) losses are

$$VaR_q(L) = E\{L \mid a_q(X)\}$$

- the q-th percentile systemic losses → q-th percentile scenario
- e.g. for M=10,000, the 99.9% loss → 10th worse scenario for factor x

- Marginal systemic risk contributions (SRC) → obligor expected losses in the q-th percentile scenario (note that expected losses are additive across obligors)

$$SRC_j = l_j \cdot p_j(a_q(X))$$
$$= l_j \cdot N\left(\frac{N^{-1}(p_j) - b\ (a_q(X))}{\sqrt{1-b^2}}\right)$$

BIS II Portfolio Model– systemic risk

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1. Scenarios
2. Conditional probs. $\boldsymbol{p}(X)$
3. Obligor (average) losses $l(X)$
4. Cond. systemic losses $\mathrm{E}\{L|\boldsymbol{X}\}$

$X = x_1$ $\quad p_j(x_1) = N\left(\frac{N^{-1}(p_j) - b\, x_1}{\sqrt{1-b^2}}\right)$ $\quad lj,\ j=1,\ldots,n$ $\quad E\{L \mid X = x_1\} = \sum_{j=1}^{n} l_j \cdot p_j\{x_1\}$

$X = x_2$ $\quad p_j(x_2) = N\left(\frac{N^{-1}(p_j) - b\, x_2}{\sqrt{1-b^2}}\right)$ $\quad lj,\ j=1,\ldots,n$ $\quad E\{L \mid X = x_2\} = \sum_{j=1}^{n} l_j \cdot p_j\{x_2\}$

$X = x3$ $\quad p_j(x_1) = N\left(\frac{N^{-1}(p_j) - b\, x_3}{\sqrt{1-b^2}}\right)$ $\quad lj,\ j=1,\ldots,n$ $\quad E\{L \mid X = x_3\} = \sum_{j=1}^{n} l_j \cdot p_j\{x_3\}$

$$\text{Regulatory Capital} = \left(\sum_{j}^{n} E_j \cdot RW_j\right) \times 8\% + GA$$

$$VaR_q(L) = \sum_{j=1}^{n} l_j \cdot p_j\{x_3\}$$

BIS II Portfolio Model – systemic risk

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BIS Calibration :

- Assume one-factor model
- For 99.9%, α = -3.09
- $b = \sqrt{R}$

$$SRC_j = l_j \cdot N\left(\frac{N^{-1}(p_j) - b\,(a_q(X))}{\sqrt{1-b^2}}\right) = l_j \cdot N\left(\frac{N^{-1}(p_j) + 3.09 \cdot \sqrt{R}}{\sqrt{1-R}}\right)$$

$$BRW(PD) = N\left((1-R)^{-0.5} N^{-1}(PD) + 3.09 \cdot (R/(1-R))^{0.5}\right) \cdot [1 + 0.0470 \cdot (1-PD)/PD^{0.44}]$$

BIS II Portfolio Model – systemic risk

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Maturity adjustment

$$BRW(PD) = N\left((1-R)^{-0.5} N^{-1}(PD) + 3.09 \cdot (R/(1-R))^{0.5}\right) \cdot [1 + 0.047 \cdot (1-PD)/PD^{0.44}]$$

- Attempts to capture additional capital against fair-value changes
- Based on 3-year maturity loan
- Derived empirically ("judgemental pooling of information")
- Maturity adjustment = 1 for one-year loans
  - product of this term with [1 + b(PD) x (M-3)] (maturity adjustment) = unit
- The explicit form of b(PD) depends on whether exposures are expressed from a MtM or purely default models

BIS II Portfolio Model – granularity adjustment

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- Objective: adjust the 99.9% systemic losses → actual losses

$$\text{Regulatory Capital} = \left(\sum_j^n E_j \cdot RW_j\right) \times 8\% + GA$$

$$RW_j(PD_j) = \min\{LGD_j \cdot BRW(PD_j) \cdot (1 + \text{b}(PD_j) \cdot (M-3)),\ 12.5 \times LGD_j\}$$

- $GA = TExGSF/n^*$ (before 4% base risk weight rebate)
  - n^* = reciprocal of Herfindahl index
  - GSF = granularity scaling factor

    $= (0.6 + 1.8\ \text{LGD}_{AG})\text{x}(9.5 + 13.75\ \text{PD}_{AG} / \text{F}_{AG})$

    - F_{AG} = systemic risk sensitivity

$$F_{AG} = N\left((1-R)^{-0.5} N^{-1}(PD) + 3.09 \cdot (R/(1-R))^{0.5}\right) - PD$$

Portfolio Credit Risk: economic & regulatory capital

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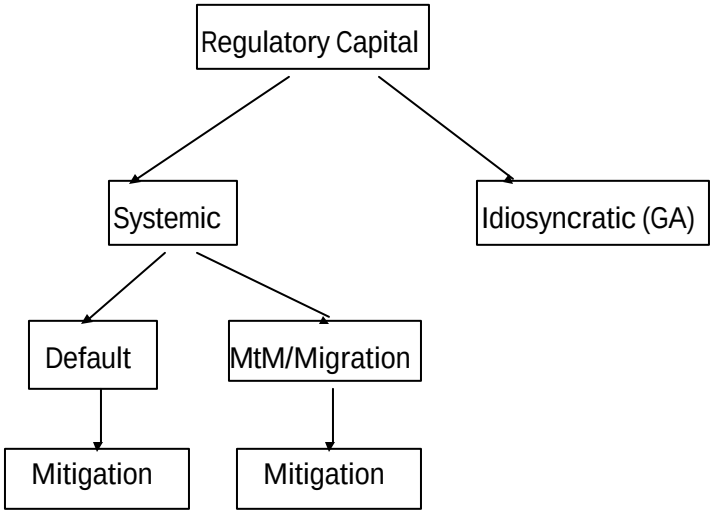

Portfolio Credit Risk: Regulatory Capital Breakdown

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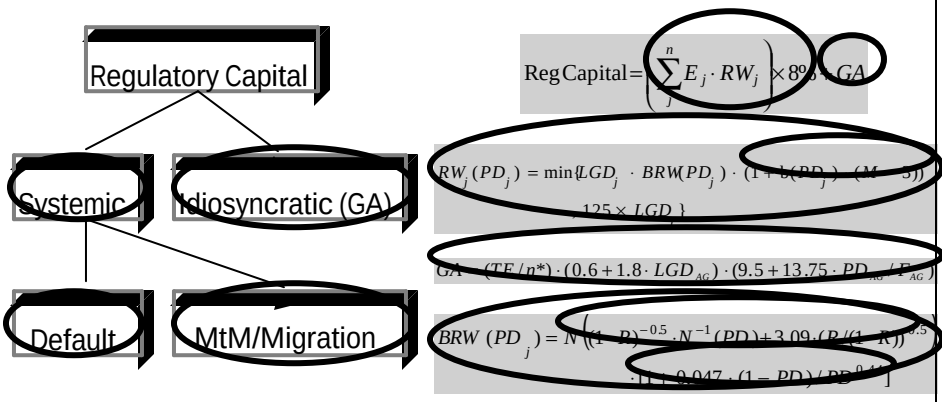

- BIS II formulae provide a transparent way to break down regulatory capital calculations.

Portfolio Credit Risk: economic & regulatory capital

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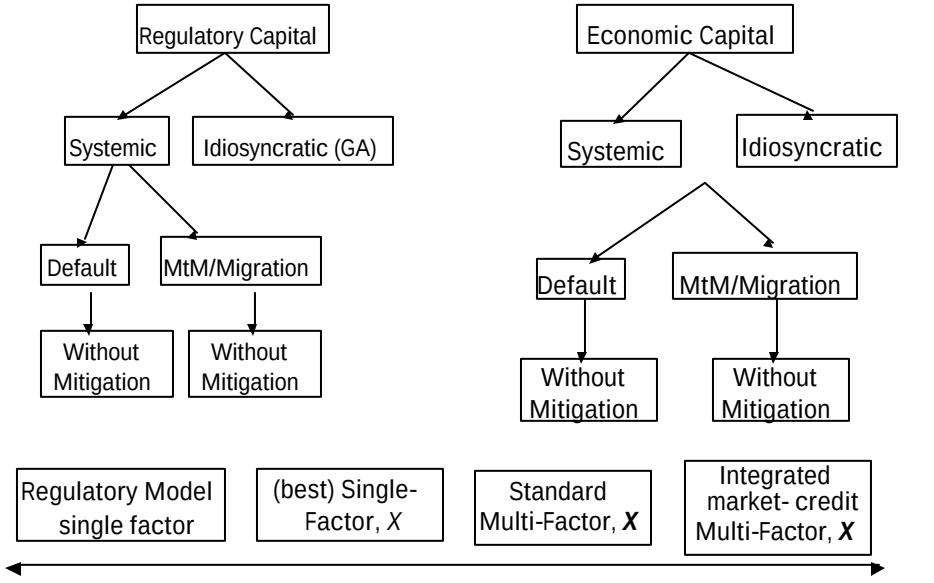

Outline

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- Introduction
  - Credit capital and BIS II
  - Credit Exposures for derivatives
- General Portfolio Credit Risk Framework
  - Integrated Market-Credit Risk
  - Portfolio Model in BIS II weights
- Case study
  - Credit Risk with Stochastic Exposures/LGDs

Motivation

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First generation PCR models are specific instances of the general framework

One limitation that first generation models share:

- Assume, in general, deterministic market factors: IRs, spreads, FX, etc.
- Exposures are not stochastic & LGDs are also generally not stochastic (more importantly not correlated)
  - May be OK for some loans & bonds (specially floating rate)
  - Not appropriate for derivatives (e.g. swap), loans with optionality, portfolios with collateral
  - Also does not capture properly risk from spread moves/volatility, or business cycle impact on LGDs

A comprehensive framework requires full integration of market and credit

- Portfolio credit risk, exposures (wrong way), specific risk for bonds...

Motivation

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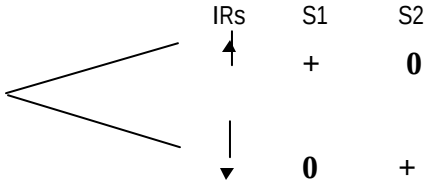
Portfolio: Two counterparties

- CP1: 1 swap - pays fix
- CP2: 1 swap - pays float
- similar maturities



- A positive MtM with both is not possible in ***any*** state of the world:
- Portfolio models generally take a single exposure number for each CP ---> cannot estimate economic capital correctly
- Question: by how much?

Stochastic Exposures/LGDs

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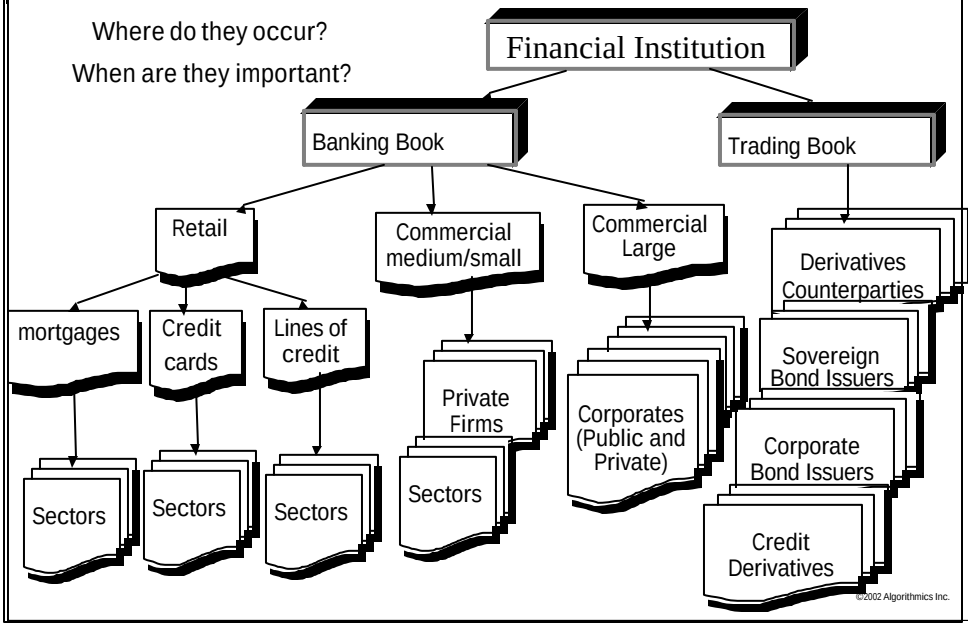
Where do they occur?

When are they important?



Case Study

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The objective: illustrate how stochastic (and correlated) exposures & LGDs affect credit capital.

Credit Losses depend mainly on four factors:

1. the volatility/dispersion of individual exposures/LGDs
2. portfolio granularity (i.e., the level of portfolio diversification)
3. market correlations (codependence of exposures/LGDs)
4. market and credit correlations

Underlying Credit Model

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- Portfolio of *m* obligors
- One-factor Vasicek model (a two-state form of CreditMetrics model)
  - same model underlying the regulatory proposal of the Basel committee
- For a given obligor, its obligor creditworthiness index in one year, *Y*, is a standard normal variable

$$Y = ß\, Z + \sqrt{1 - ß^2}\, e$$

- Single systemic factor Z ~ N (0, 1)
- Specific component ε ~ N (0, 1)

Portfolio Model

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- Unconditional probability of default

$$\tilde{p} = \Pr\{Y < a\} = \Phi(a)$$

- Default probability of an obligor *conditional* on a given scenario (the systemic factor) is:

$$p(Z) = \Phi\left(\frac{\Phi^{-1}(\tilde{p}) - b\,Z}{\sqrt{1 - b^2}}\right)$$

Portfolio Model

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- Consider various homogeneous portfolios
  - obligors are statistically identical (the same weight β, and exposure)
- β^2 is the systemic component of the creditworthiness variance
  - c*redit correlation* or the "asset correlation"
- Losses from a given obligor can be stochastic
  - *V* (X), where X is a vector of "market" state variables
- A *scenario*, or state of the world: joint outcome *(X, Z)*
  - *(X, Z)* follow a joint normal distribution

Instruments, Credit & Market Data

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Consider various homogeneous portfolios where:

- All counterparties are rated BB (one-year default probability of 1%)
- Each counterparty contains a single position in one of two instruments:
  - a fixed receiver or a payer USD interest-rate swap
  - the coupon on the fixed leg is 6.636
- LGD = 100%

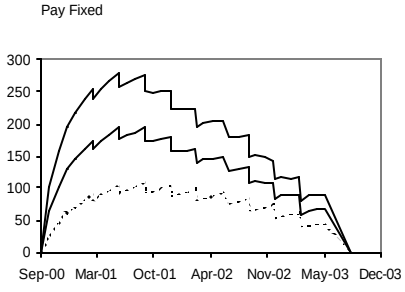
- Swap exposure *V(X)* represents the losses conditional on a given scenario *(X, Z)*
- Interest rates follow a multi-factor, mean-reverting process
- Denote by ρ is the *market-credit correlation*
  - the negative of correlation of the single credit driver, Z, and the first principal factor of the model for interest-rates evolution through time

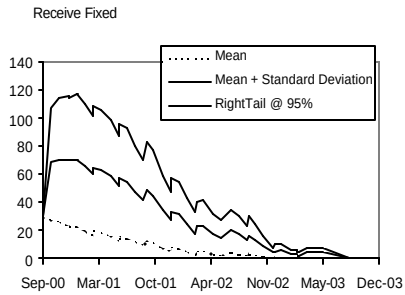
Computation

Simulation:

- Draw a large sample of scenarios (*X, Z)* from their joint distribution
- Under each Scenario:
  - compute the conditional (systemic) default losses for each obligor *V(X)*
  - compute the conditional default probabilities for each obligor *p(Z)*
  - compute the *conditional portfolio loss distribution*
    - conditional defaults are now independent events
- Obtain t he *unconditional* portfolio loss distributions by averaging over all scenarios.
- Systemic risk of the portfolio by applying the Law of Large Numbers (LLN)

Swap Exposures Profiles (USD)





	Pay Fixed	Receive Fixed
Expected Exposure	89	13
Standard Deviation	77	35
Quantile (95%)	228	89
Quantile (99%)	288	167
Quantile (99.9%)	368	265

Credit Capital for Finite Portfolio

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Two Models:

Deterministic exposures (DE):

- Loan-equivalent exposures used as model inputs = expected exposures

Stochastic exposure (SE):

- Calculated through the Monte Carlo simulation

Note: to capture losses over one year, a multi-step portfolio model is required.

- precise timing of default during the 1y horizon can have substantial impact
- we keep the problem simple to focus on the impact of exposure volatilities and correlations by using a single-step model.

Case Study

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- Base Case
  - Finite portfolio of payer swaps
  - What happens for a similar portfolio of receiver swaps?
- Impact of credit correlations
- Impact of market-credit correlations
- Market hedged portfolio
- Impact of portfolio size
  - systemic losses

Base Case: Loss Statistics

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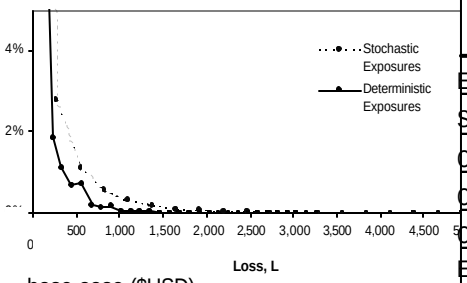
• Homogeneous portfolio of 72 counterparties, each with a payer swap.



base case ($USD)

	Deterministic Exposures	Stochastic Exposures
Expected Losses	64	77
Standard Deviation	137	229
Credit VaR (95%)	292	347
Credit VaR (99%)	558	1,018
Credit VaR (99.9%)	1,181	2,317
Expected Shortfall (95%)	421	852
Expected Shortfall (99%)	729	1,666
Expected Shortfall (99.9%)	1,420	3,275

(72 payer swaps, $\beta^2 = 0.25$, $\rho = 0.25$)

Tails of Portfolio Loss Distributions

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Some comments:

- The tail of the deterministic case is truncated at losses of about $6,400
  - Maximum possible losses when all 72 swaps default for a loss of $89 each
- With stochastic exposures such a loss could occur when only about one-quarter of the swaps default on an extreme market move
  - Alternatively: the joint event of having a 99.9% market move and all counterparties default, would produce losses over four times larger.

Portfolio of receiver swaps

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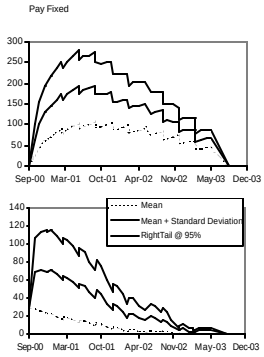

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Expected Exposure	89	13
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- While some loss statistics are over 200% higher for a portfolio of payer swaps, the percent *difference in the models is much larger for a similar portfolio of receiver swaps*
  - relative difference between mean and tail-swap exposures is much larger for receiver swap; e.g. 99.9% tail exposure is over 20 times the expected exposure

Impact of Credit Correlations

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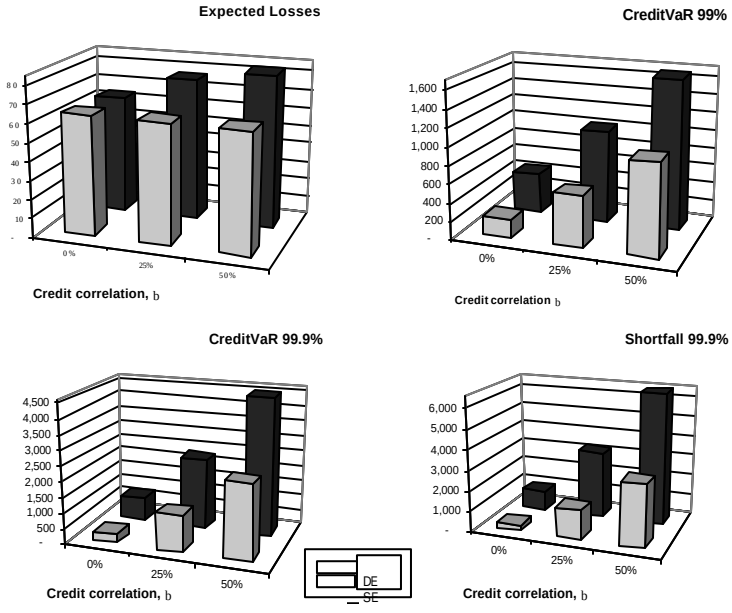

Impact of Market-Credit Correlation

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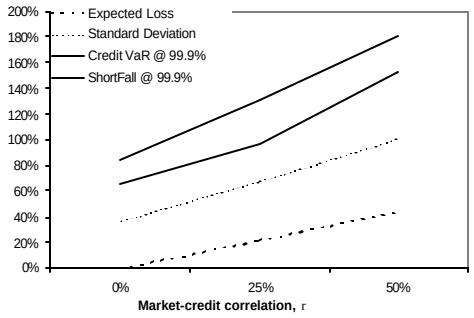
- If market & credit risks are uncorrelated
  - no impact on expected losses, but
  - the impact on dispersion measures is substantial (over 40% for CreditVaR and 80% for expected shortfall)



- Model differences increase dramatically with correlations; e.g CreditVaR almost triples at a 50% corr., and expected losses are already 40% higher
- The accurate modelling of stochastic exposures and correlations has a substantial impact on both credit reserves and economic capital

Market-Hedged Portfolio

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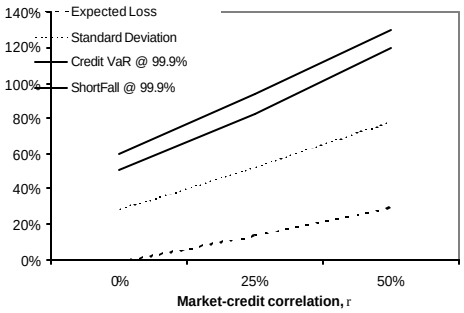
- Portfolio with 72 counterparties, half of which contain a single payer swap, and the second half a signle receiver swap.
  - Portfolio that is hedged for market movements and does not contain, as a whole, market risk
  - Possible to have simultaneous defaults of more than one half of the counterparties, regardless of the state of the world.

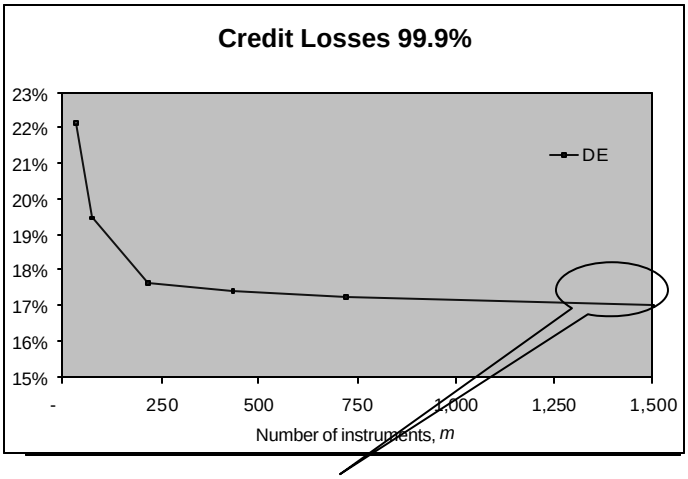


Market-Hedged Portfolio

- It is not easy to predict the results in this case because there are several effects moving losses in different directions.
  - "Market hedges" tend to diminish the loss probabilities in any given scenario
  - Losses may be incurred in almost all scenarios since one of the two swaps will be in the money
  - The receiver swaps in the portfolio increases the model differences in this case since, as was explained earlier, the percent difference between the expected and tail exposures is much larger for these instruments

Portfolio Granularity and Stochastic exposures/LGD

- Deterministic Exposures



- Law of Large Numbers: Infinitely granular portfolio distribution of Losses → distribution of expected losses

Portfolio Granularity and Stochastic exposures/LGD

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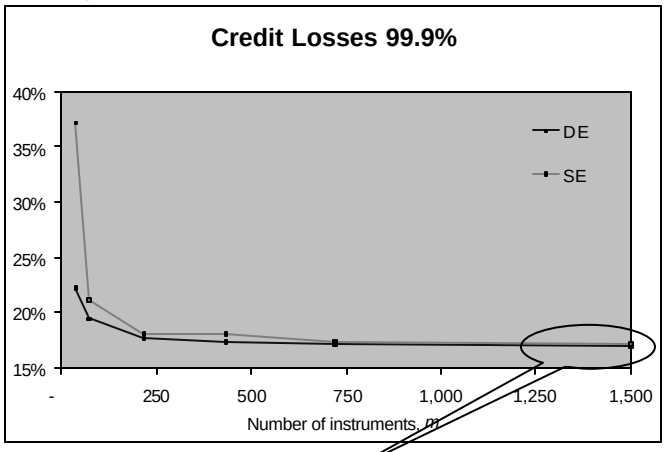
- Stochastic Exposures – No correlations



- For large portfolios Exposures/LGD volatility is unimportant if these are uncorrelated

Portfolio Granularity and Stochastic exposures/LGD

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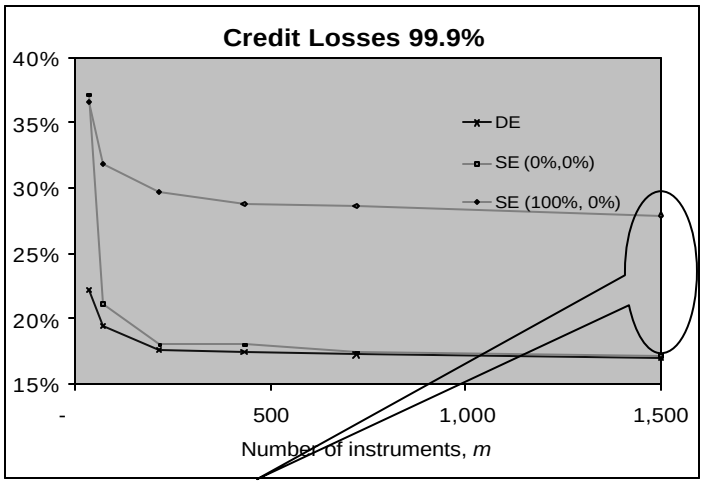
- Correlated Exposures – uncorrelated market and credit



- Exposure/LGD Correlations may contribute substantially to Losses even for large portfolios

Portfolio Granularity and Stochastic exposures/LGD

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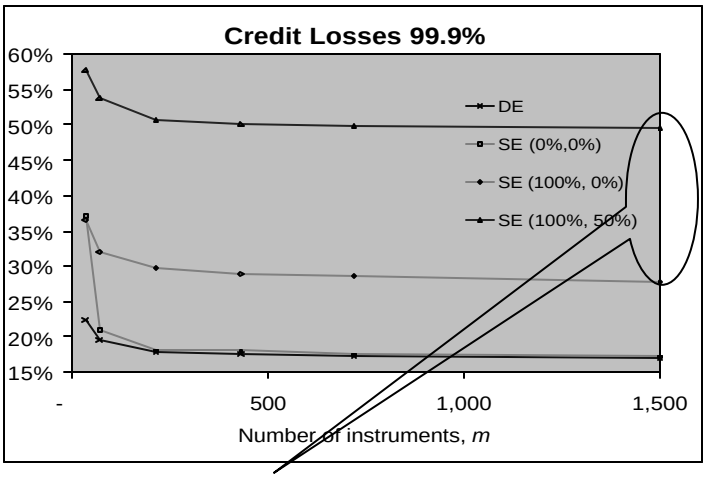
- Correlated Exposures – Correlated market and credit



- Market and credit correlations may contribute substantially to credit losses

Portfolio Granularity and ~~Stochastic exposures/LGDb~~

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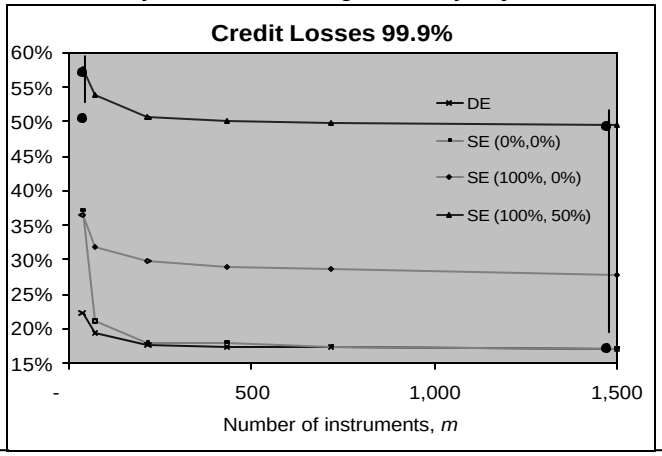
- Implications for systemic risk and granularity adjustment



- ~~"Adjustment" to systemic risk for: correlations and wrong way exposures~~
  - GA should also correct for volatility and correlations

Case Study
Concluding Remarks

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- Stylized case study: demonstrates the impact of stochastic and correlated exposures on credit capital.
  - Main results and implications apply as well when LGDs are stochastic.
- Four factors have a substantial effect on credit losses:
  - exposure (market) volatility, credit correlations, market-credit correlations, portfolio granularity.
- Results have important implications for both economic and regulatory Capital
  - Stochastic exposures and their correlations cannot be ignored even for infinite granular portfolios and systemic losses.

Concluding Remarks

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- Short of allowing full internal portfolio credit-risk models, two alternatives can correct for this in the current regulatory proposal
  - add a "systemic correlation adjustment" at the portfolio level (similar to GA).
    - internal models for LEEs can be based on expected exposure
  - LEEs computed directly from an internal model that corrects for correlations
- Analytical solutions might be possible for some simple cases only; otherwise, a simulation model can be used.

- Substantial benefits in using a flexible, integrated market and credit-simulation model for portfolios with stochastic exposures and LGDs, spreads and portfolio collateral, and other mitigation techniques.

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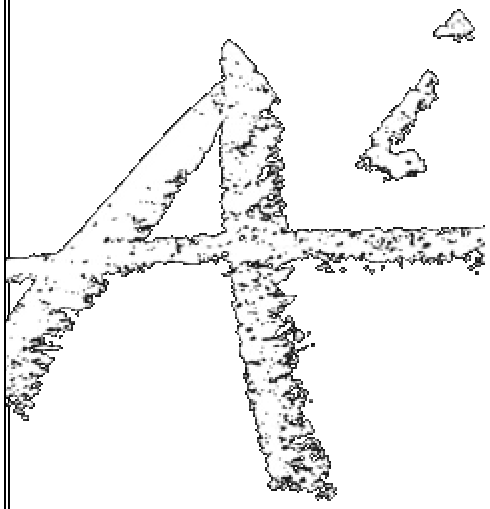

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- Algo Research Quarterly