Embedded Options in Insurance Contracts: Guaranteed Annuity Options

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Outline

- Introduction
- Background UK insurance products
- Description of the embedded option
- Factors that affect the value of the GAO
- Time series analysis of these factors
- Maturity value of the option over time
- Methods of managing the risk
- Option pricing approach
- Lessons

Products with Investment Guarantees

- Worldwide trend to offer sophisticated features in retail products.
- These products increasingly sold by banks, financial institutions and insurance companies
- Provide range of benefits and options to the consumer.
- For example the customer can benefit from upside appreciation in the equity market and have a floor level of basic protection.
- In the USA equity indexed annuities include a rich variety of options : eg they can include Asian features and lookback features

Three Examples

- *Death Benefits under Variable Annuities*
- On September 4, 2002, Wall Street Journal reported that Cigna had to take a USD 720 million charge to cover its liabilities under this business
- *Insurance Benefits under Canadian Segregated Funds*
- Described in 1999 by Risk Magazine as Canada's option nightmare. Now under better control
- *Guaranteed Annuity Options under UK pension contracts*
- Put a severe strain on several UK companies

UK : With Profits Policies

- With profits policies.
- Consumer pays premiums to insurer to fund the maturity sum assured.
- Reversionary bonuses declared annually but *payable* at maturity. Size of bonus related to investment performance. Smoothing goes on.
- At maturity a terminal bonus payable. Amount depends on investment performance.
- UK insurers invest heavily in the stock market. Returns on with profits policies are strongly related to stock market performance.

Unit Linked Contracts

- In this case premiums invested directly in a reference portfolio(often equities)
- Investment gains and losses passed directly to the consumer.
- Mutual fund plus insurance: sometimes tax advantages.
- In Canada these contracts are called segregated fund contracts. In the USA variable annuities.
- Under a unit linked contract maturity amount directly linked to the performance of the reference portfolio.
- Good returns when the stock market performs well.

Impact of Interest Rates

- Insurance polices widely used to fund retirement income. At maturity (65 males) policy proceeds converted into life annuities.
- Amount of annuity depends on proceeds available and the rate of conversion of lump sum into annuity.
- Assume retiree lives exactly 13 years after retirement. Then the annual payment a is given by

$$1000 = a \sum_{j=1}^{13} \frac{1}{(1+i)^j}$$

  where i is the rate of interest.
- As interest rates rise a increases.

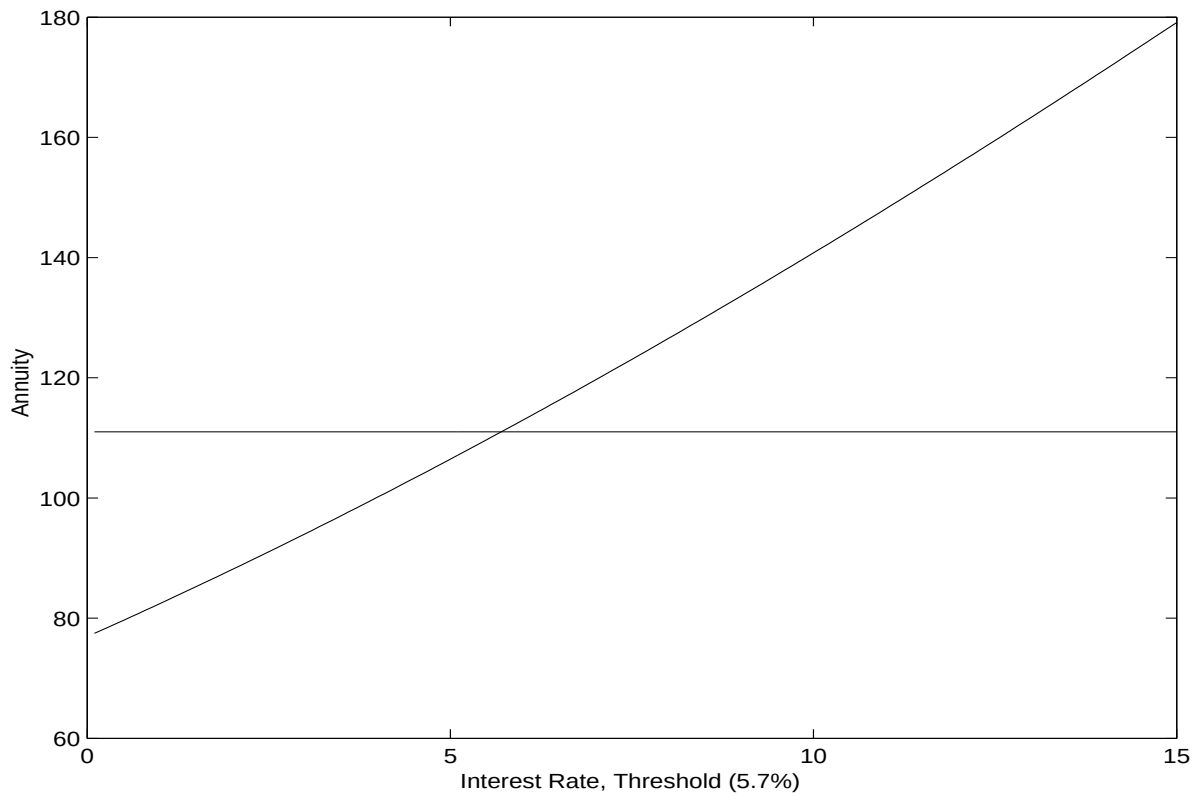

Figure 1: Amount of annuity purchased by $1,000$ as interest rate varies .

The Guarantee

- Many pension policies issued in the 1970's and 1980's contained a guarantee.
- Common guaranteed annuity rate for male at age 65 was 9. Each 1000 of proceeds was guaranteed to provide 111 annual pension. Payable during retirement.
- This guarantee was issued when long term interest rates were very high
- This option becomes more valuable if interest rates fall.
- Initially the option was well out of the money. It was assumed to have zero cost. This liability was *totally neglected* by insurance companies until mid 1990's
- This liability threatened solvency of some UK insurers. Brought down the venerable Equitable Life

Decline and Fall of long rates

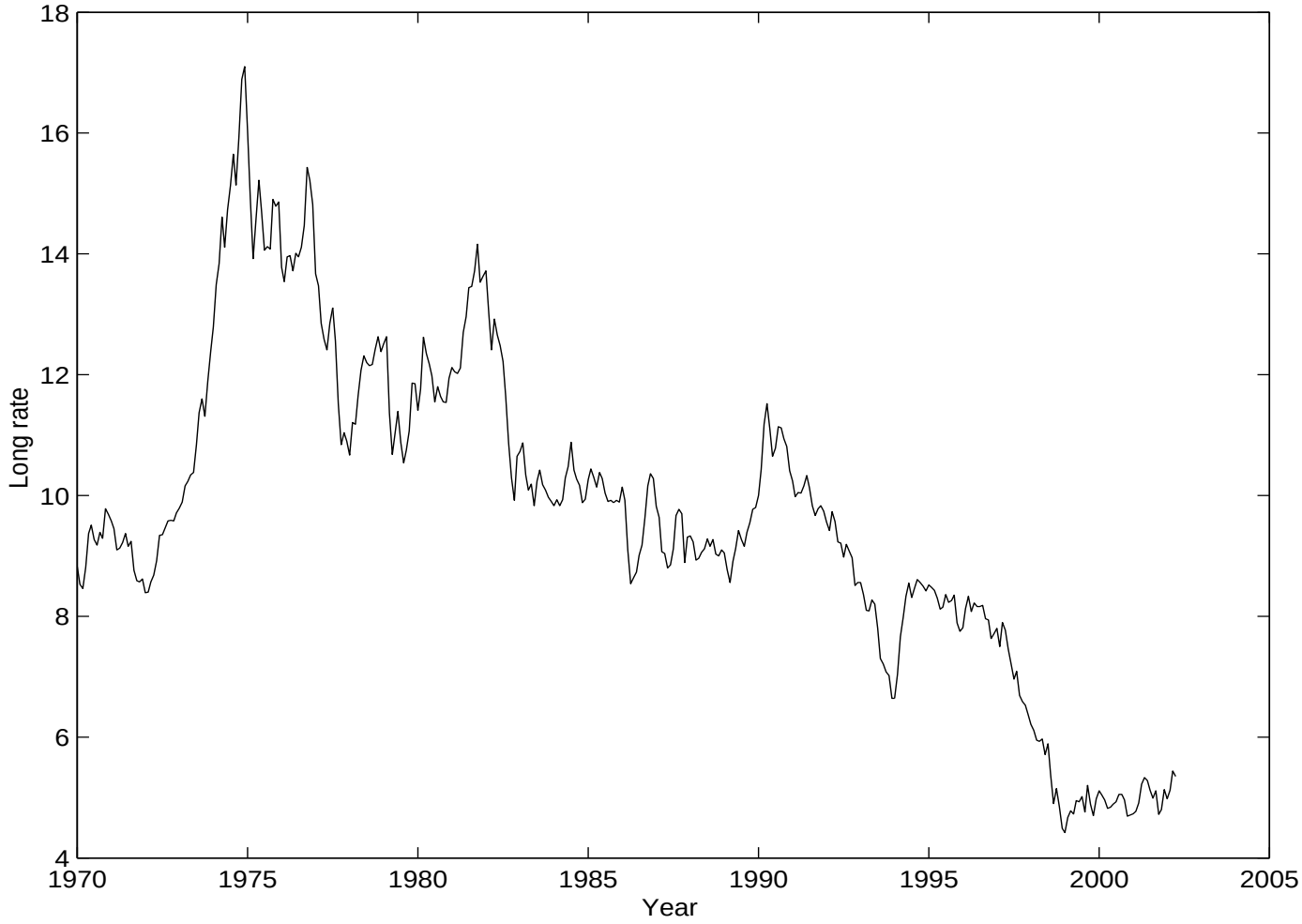

Figure 2: Long UK interest rates 1970-2002

Mortality Improvement

- If life expectancy increases and people live longer life annuities become more expensive.
- To illustrate suppose that retiree at age 65 now will live 16 years rather than 13 years.
- The annuity amount,a, corresponding to each 1000 proceeds is given by

$$1000 = a \sum_{j=1}^{16} \frac{1}{(1+i)^j}$$

  where i is the rate of interest.
- In this case the break even interest rate is 7.7%.
- Guarantee becomes effective if interest rate falls below 7.7%. Option becomes in the money when $i < 7.7$.

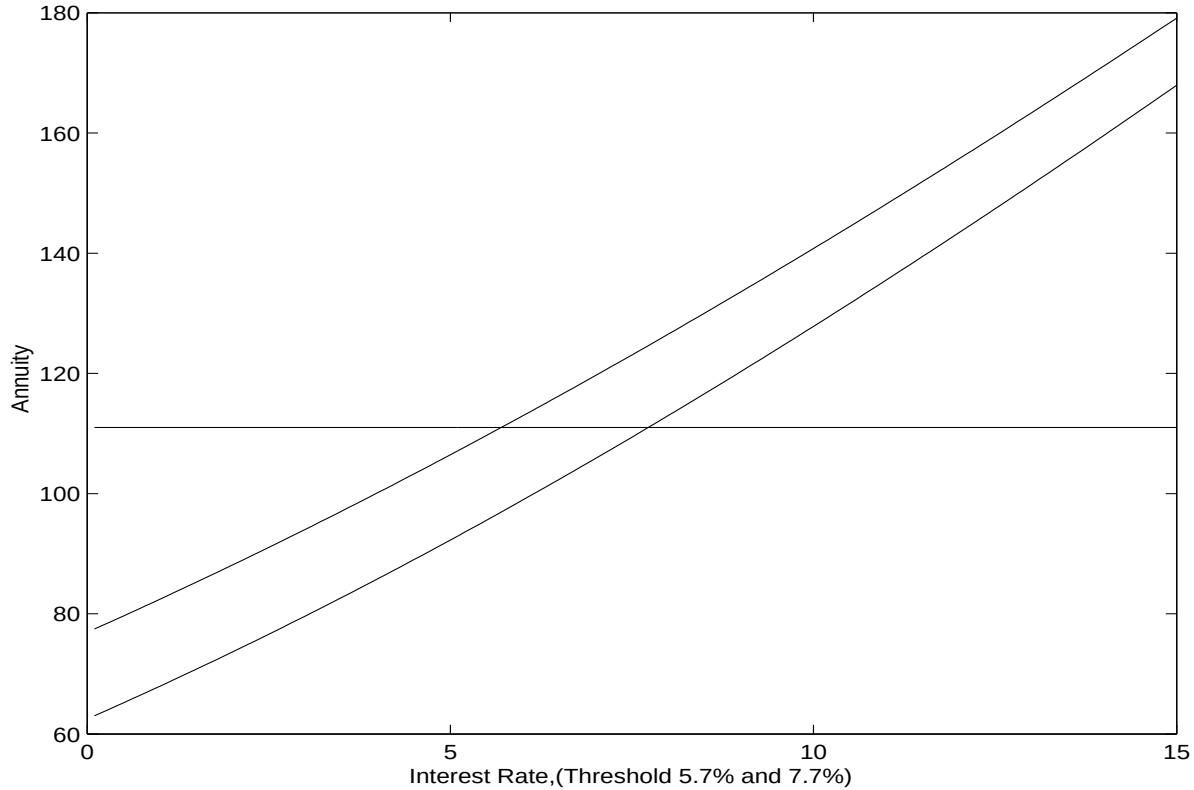

Figure 3: Amount of annuity purchased by $1,000$ as interest rate varies. Terms 13 years and 16 years

Maturity Value of GAO

- Assume$S(T)$ is the amount of the proceeds at time T and $a_{65}(T)$ is market annuity rate at time T for a life then aged 65.
- The maturity value of the guarantee at maturity(time T) for the benchmark contract is

$$S(T)\ \max\left[\ (\frac{a_{65}(T)}{9} - 1),\ 0\right] \tag{1}$$

- Note that if $S(T) = 1000$, this can be written

$$1000\ \max\left[\ (\frac{a_{65}(T)}{9} - 1),\ 0\right] = \ \max[\ (111a_{65}(T) - 1000),\ 0]$$

Three Mortality tables

- Table known as a(55) was being used to compute annuity values in the 1970's.
- The PMA80(C10) table is based on UK experience for the period 1979-1982 and projected to 2010 to reflect mortality improvements.
- The PMA92(C20) table is based on UK experience for the period 1991-1994 and projected to 2020 to reflect mortality improvements.

Table	Expectation of life	Critical interest rate
a(55)	14.3	5.6
PMA80(C10)	16.9	7.3
PMA92(C20)	19.8	8.2

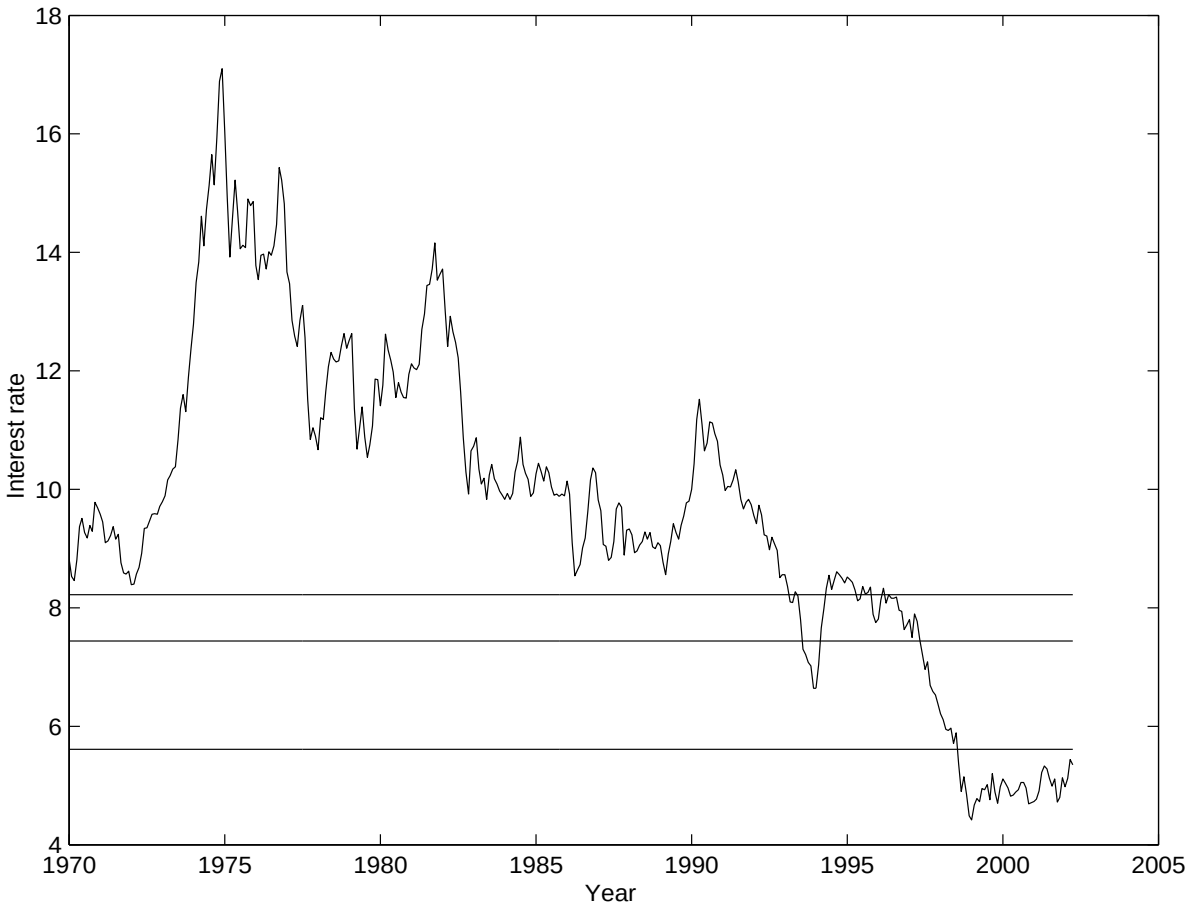

Figure 4: Interest rates levels that trigger the guarantee. Bottom line(5.6) Middle line(7.3) Top line(8.2)

Dependence on Equity returns

- Recall that the maturity value of the guarantee at maturity is proportional to $S(T)$ the maturity proceeds.
- Over the period 1980-2000 the compound return on the the major UK Index(FTSE) was 18% per annum
- Returns on with profits business consequently quite high
- Returns on unit linked business also high but also more volatile.
- These good returns served to increase the value of the option which was already in the money because of falling interest rates and improving mortality.

Value of maturing guarantee

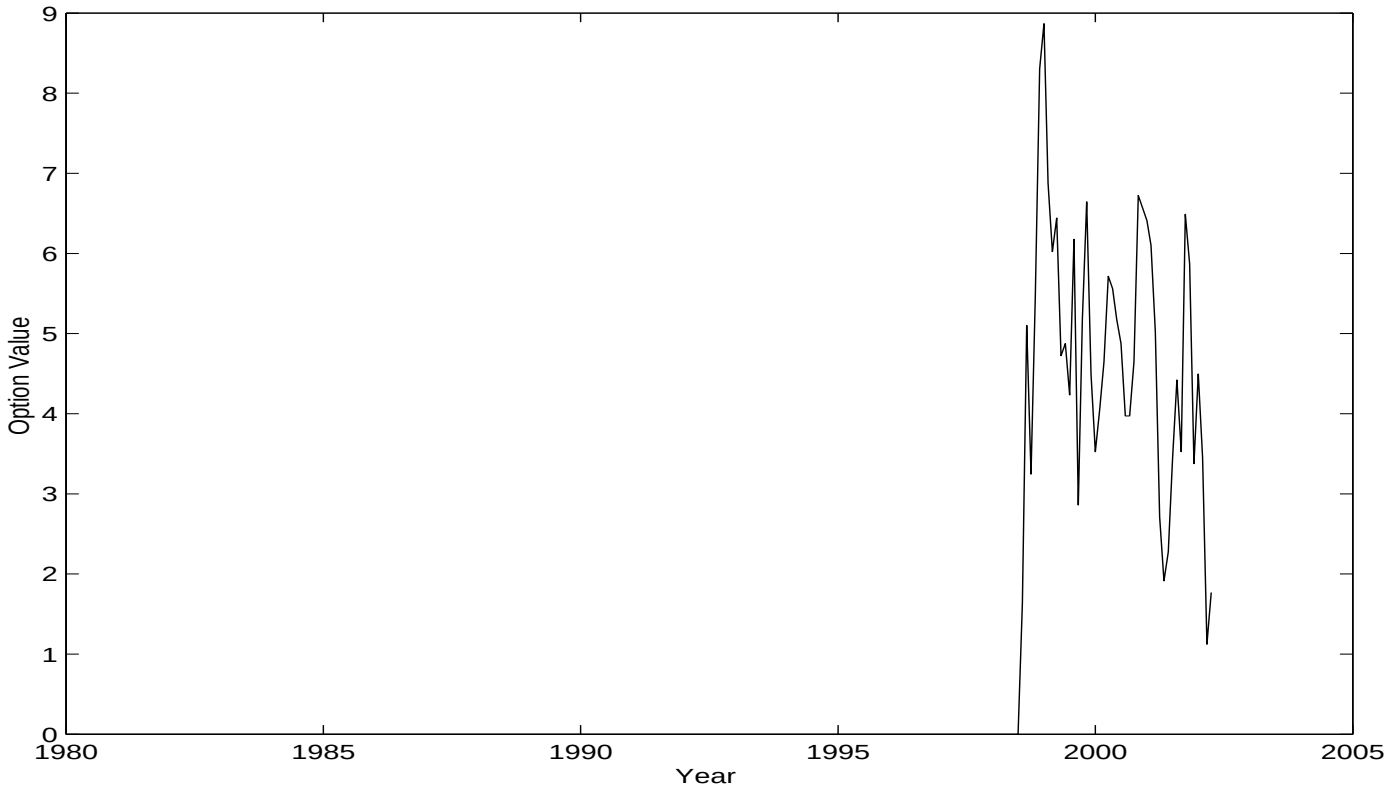

Figure 5: Value of maturing guarantee based on 100 at maturity and a(55) mortality. Threshold rate 5.6%.

Value of GAO *at Maturity*

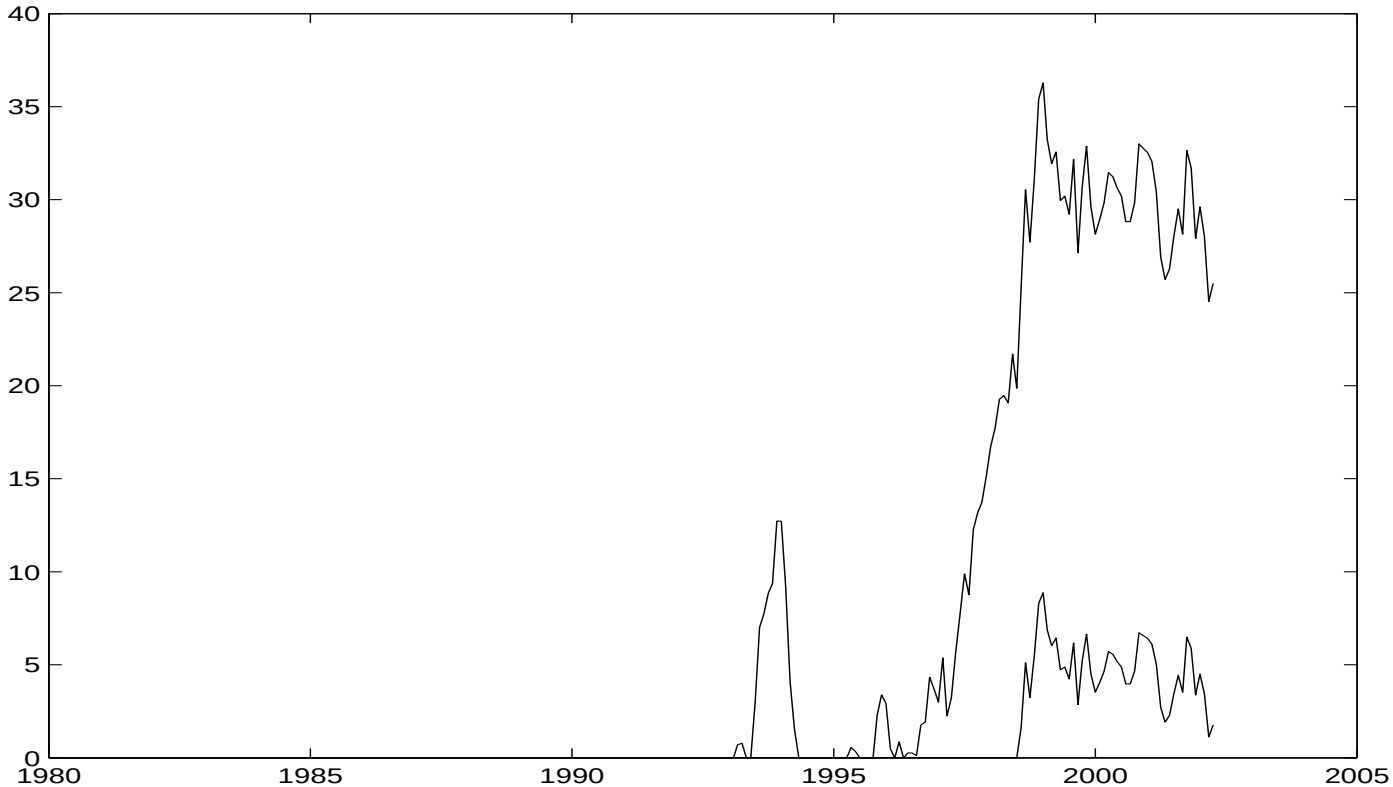

Figure 6: Value of maturing guarantee based on both 100 under two mortality assumptions: a(55) and PMA92(C20) mortality.

The Story so far

- Options written on these contracts were far out of the money at inception and deemed to have zero cost.
- Value of option increase with
  1. Fall in interest rates
  2. People living longer
  3. Good equity returns
  4. Public awareness of option's existence.
- As it turned out all four of these occurred. How was the risk managed?

Some history and questions

- Liability for these options was not considered when they were written and ignored until mid 1990's
- Emerging liability was not recognized and therefore not disclosed
- These are complex long term options that are difficult to hedge
- However it should have been possible to price them from the late 1980's onward.
- The technology was available in the public domain
- Professional conservatism: resistance to *new fangled ideas*

Ways to Manage the Risk

- Three methods
- First keep a separate pool of capital
- Insurer sets aside additional capital(reserves) to ensure that the liabilities under the guarantee will be covered with a high probability.
- The liabilities are estimated using a stochastic simulation approach. The basic idea is to simulate the future using a stochastic model. VaR idea.
- These simulations can be used to estimate the distribution of the cost of the guarantee.
- From this distribution one can compute the amount of initial reserve so that the provision will be adequate say 99% of the time.

Quantile reserving

Even if contingency reserves at, say, a 1 in 1,000 level had been set up at the end of 1984 for these policies (and the methodology was known by that time), these reserves might not have been enough. But their insufficiency would have emerged as interest rates fell and mortality rates improved during the 1990s, giving the society time to remedy the situation.

David Wilkie letter to the Actuary, November 2000

Reinsurance

- The second approach is to reinsure the liability with another financial institution such as a reinsurance company or an investment bank.
- Insurer buys corresponding option from an investment bank.
- Scottish Widows bought structured product from Morgan Stanley (One and one half billion pounds sterling)
- When this method used concerns about credit risk of the intermediary.

Dynamic Hedging

- Uses basic ideas from option pricing
- Set up an initial portfolio of tradeable securities .
- Get the weights from the option formula
- Adjust (or dynamically hedge) this portfolio over time.
- This portfolio is selected to replicate the option payoff at maturity.
- In the GAO situation there are several challenges in using this approach.

Another Method to reduce the Liability

- Recall that the liability is proportional to $S(T)$ maturity value of the policy.
- One way to reduce liability for with profits policy: control the size of $S(T)$.
- Equitable Life in 1993 proposed to reduced the terminal bonus on its GAO contracts.
- This was subject to considerable legal action. Eventually settled by a ruling from the House of Lords

Stochastic Interest rate Models

- Start with a one factor interest rate model to price the guarantee
- Due to Vasicek(1977)
- Assume short term interest rate, r(t), follows a diffusion process.
- We assume

$$dr(t) = \kappa(\theta - r(t))dt + \sigma dW(t)$$

where $\kappa, \theta,\ \sigma_v$ are constants and $W(t)$ a standard Brownian Motion.

Short rate dynamics

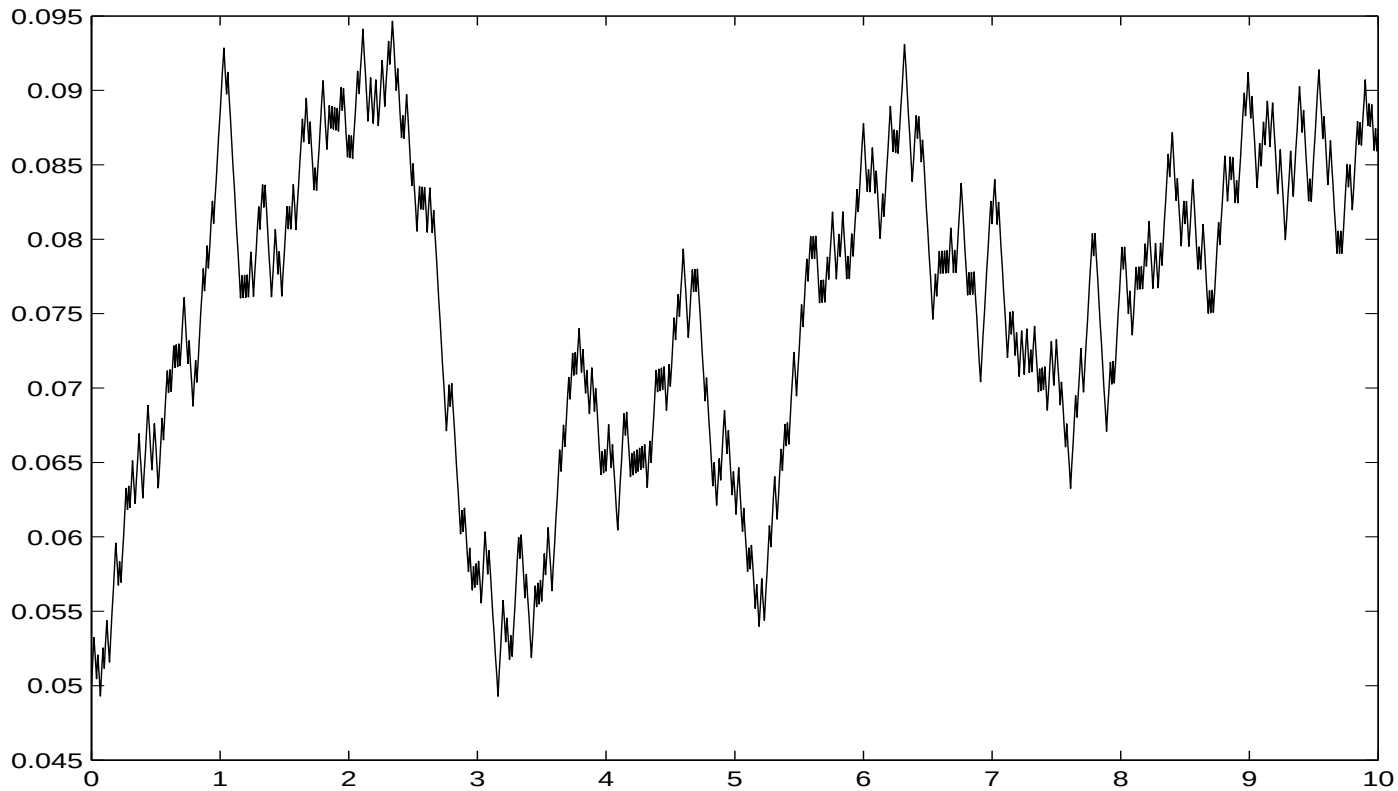

Figure 7: Short rate simulation for ten years. $r(0) = .05, \theta = 0.08, \kappa = 0.4, \sigma = .015$.

Valuation under Stochastic Interest rate Model

- Convenient to define money market account

$$B(t) = exp(\int_0^t r(u)du)$$

- Then price of an European derivative with maturity payment $V(T)$ is given by

$$\frac{V(t)}{B(t)} = E_Q\left[\frac{V(T)}{B(T)}\right]$$

- Can use to find current price of a zero coupon bond that pays one unit at time T. We get

$$D(t,T) = exp(A(\tau) - r(t)B(\tau))$$

where $\tau = T - t$ and A and B are non stochastic functions of $\tau, \kappa, \theta, \sigma$.

Options on Bonds

- Brennan and Schwartz(UBC) early 1980's
- Late 1980's closed form solution for option on a zero coupon bond in the Vasicek model Jamshidian(1989), Sharp(1987) Chaplin(1987)
- Consider a European call option on a $(T+j)$ maturity bond. Option matures at time T. Strike price is K. The formula for the call option under the Vasicek model is

$$Call = \; D(t, T+j)\, N(h_1) \; - \; K D(t,T)\, N(h_2)$$

where

$$h_1 = \frac{log \frac{D(t,T+j)}{D(t,T)K}}{\sigma_P} \; + \; \frac{\sigma_P}{2},$$

$$h_2 = \frac{log \frac{D(t,T+j)}{D(,t,T)K}}{\sigma_P} \; - \; \frac{\sigma_P}{2},$$

and

$$\sigma_P \;=\; \sigma \sqrt{\frac{1-e^{-2\kappa(T-t)}}{2\kappa}}\;\frac{(1-e^{-\kappa j})}{\kappa}.$$

GAO as Option on coupon bond

- The guaranteed annuity option can be viewed as an option on a coupon paying bond. The coupons are adjusted by the survival probabilities.
- Jamshidian (1989) showed that we can get a closed form solution for an option on a coupon paying bond in any one factor model.
- We use these ideas to value interest rate option in the GAO assuming we can predict the mortality correctly.
- We give the formula on the next slide.

GAO Option Formula

$$G(t) = \frac{{}_{T-t}p_x \; S(t)}{g} \; \frac{\sum_{j=1}^{J} a_j \, C[D(t, T+j), K_j, t]}{D(t, T)}. \tag{2}$$

Here $G(t)$ is GAO price at current time t for a life aged currently aged x. Policy matures at time T when life will be aged $R = x + (T - t)$. The symbol ${}_{T-t}p_x$ is probability of survival to retirement(maturity) and a_j are survival probabilities beyond retirement. Recall $D(t, T)$ is zero coupon bond price. We assume stocks and interest rates are independent and $S(t)$ is current portfolio value.

GAO Option Formula

- We need to make another adjustment.
- Classical Vasicek model with three constant parameters will not fit the input term structures.
- If model does not price the bonds at market little hope of sensible option prices.
- Calibration. We use approach of Dybvig(1988). Very similar to Hull White(1990)

Time series of GAO market Values

- Use Vasicek, Jamshidian and Dybvig to price GAO over time.
- Parameters

Parameter	Value
κ	0.35
θ	0.08
σ	0.025

- Volatility is the most critical Parameters based on those in the literature on UK data for roughly this time period. See Nowman(1997) and Yu and Phillips(2001).
- Contract with ten years to go. Assume stock return independent of interest rates

Time series of GAO market Values

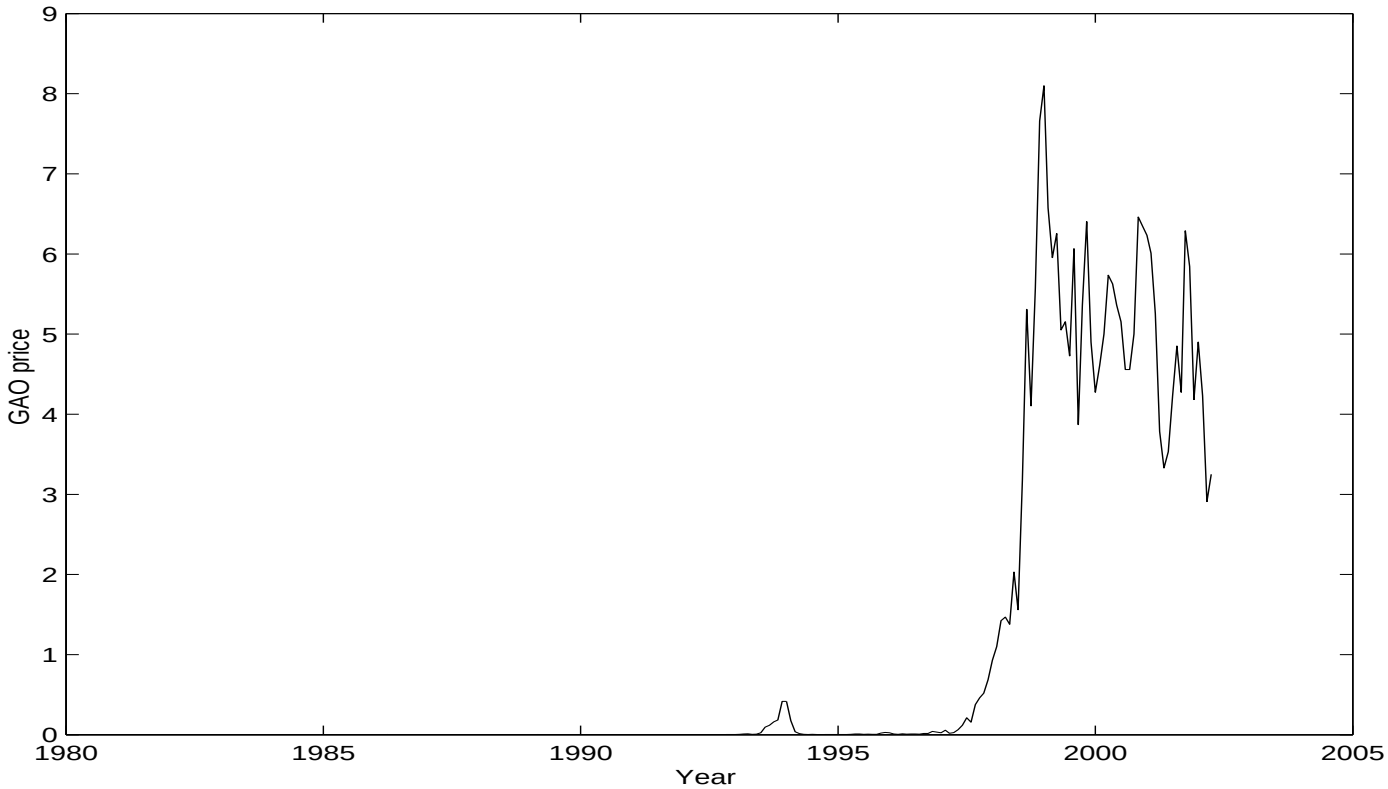

Figure 8: Market Values of GAO. Time series for male aged 55, based a(55). $\kappa = 0.35,\ \theta = 0.08,\ \sigma = .025$

Time series of GAO market Values

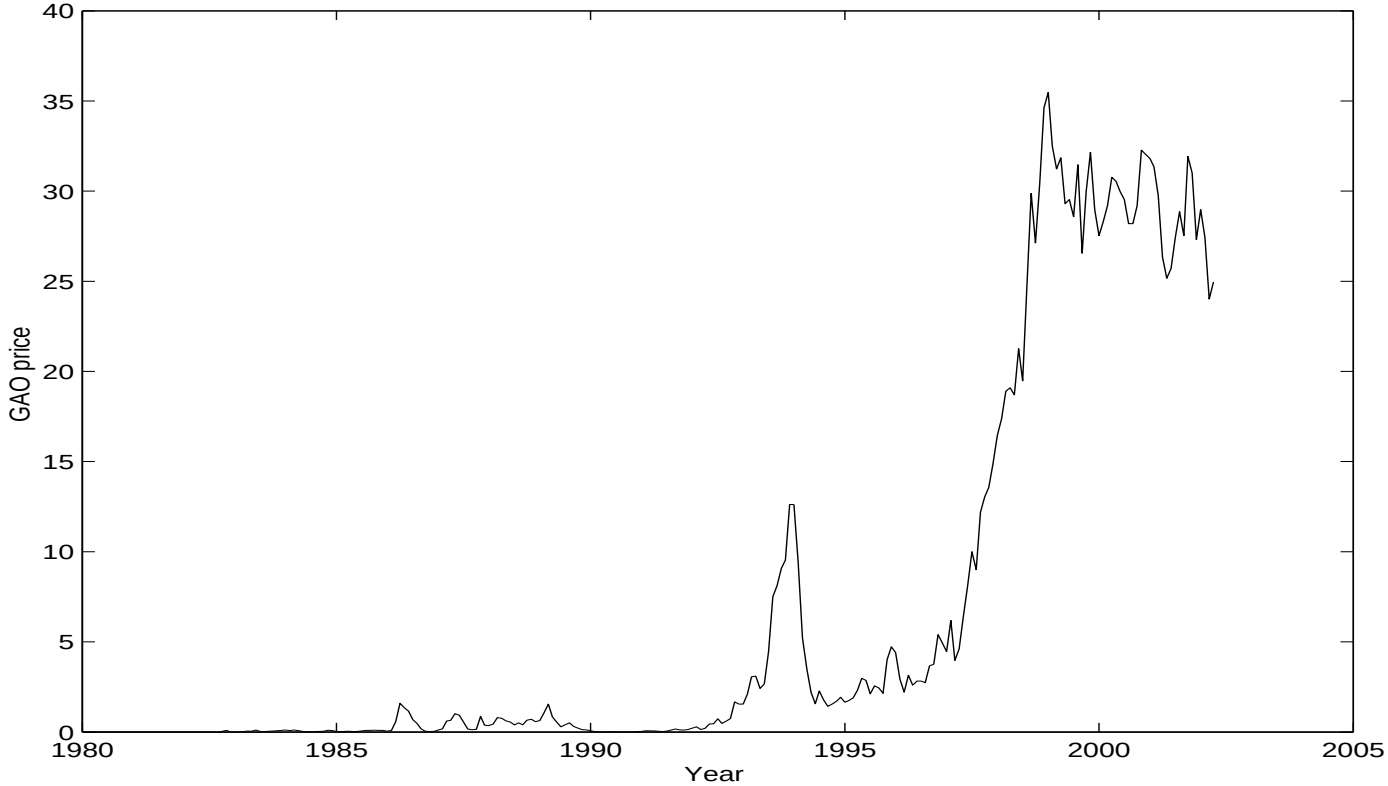

Figure 9: Market Values of GAO based on PMA92(C20).Time series for male aged 55, $\kappa = 0.35,\ \theta = 0.08,\ \sigma = .025$

Time series of GAO market Values

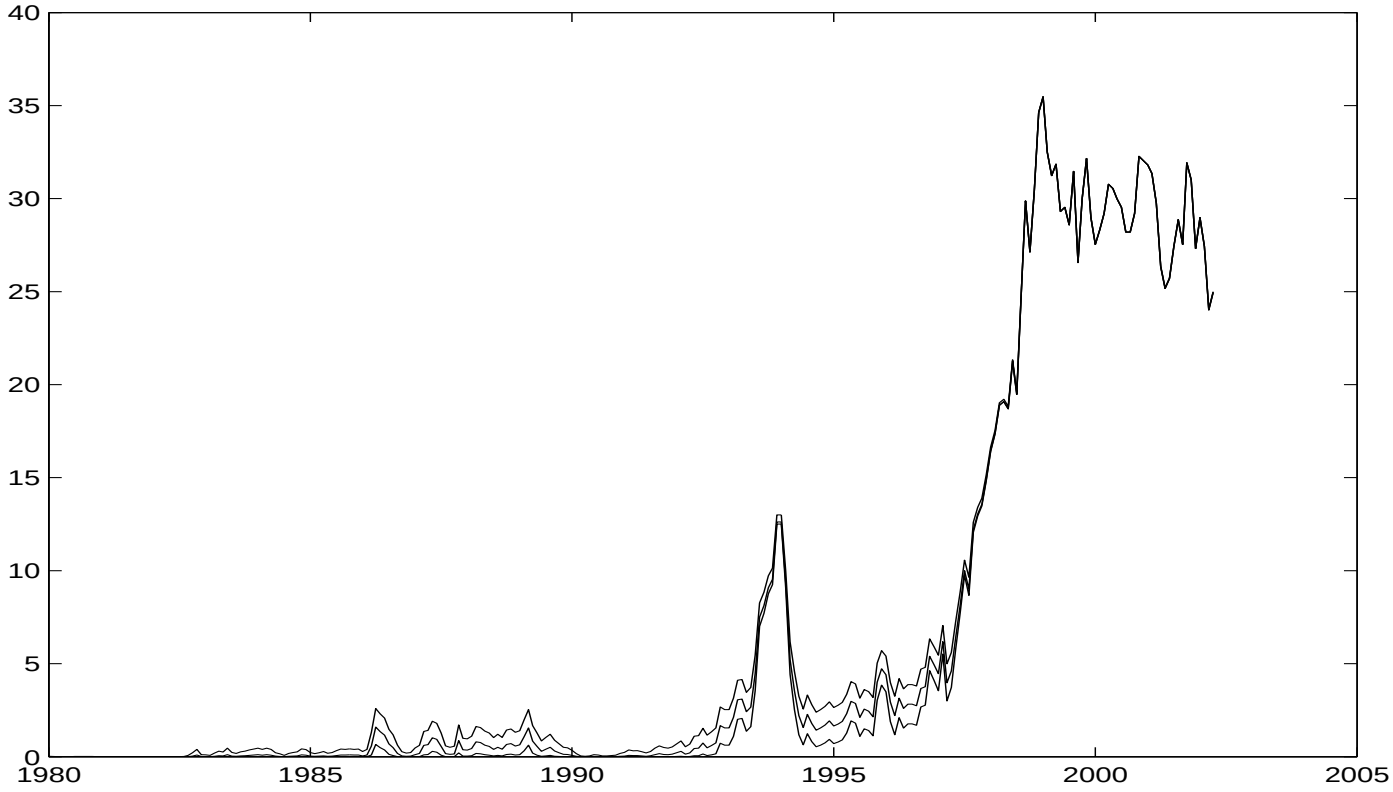

Figure 10: Market Values of GAO based on 3 volatility assumptions. These are $\sigma = 0.015, 0.025, 0.035$. $\kappa = 0.35$, $\theta = 0.08$. PMA92(C20) mortality

Hedging

- First the ideal case. Perfect frictionless markets. No arbitrage. Continuous trading
- Assume market is complete. Enough independent traded securities
- Current price of any option can be expressed in terms of portfolio of traded securities(eg Black Scholes)
- This formula gives the recipe for replicating the option payoff at maturity
- Adjust the portfolio over time in light of asset price changes

Hedging is Hard

- Many market imperfections
- It is not possible to rebalance the replicating portfolio on a continuous basis. Instead it has to be rebalanced at discrete intervals.
- The asset price dynamics may be incorrectly specified.
- Calibration is very damaging(fatal) to hedging
- In the case of interest rates seems like we need three factors. Also over long periods regime switching models required Hardy(2002)(2003)
- The GAO involves a very long term option that involves *interest rates stock returns and mortality*. Each one tricky on its own

Some Conclusions

- Insurers gave away *for free* a very complicated long term option with an open ended liability.
- It should never have been written in the first place.
- Once the options were written they should have been monitored and liabilities disclosed.
- Liability could be partly hedged by insurer taking a long position in long dated receiver swaptions.
- Responsibility for this disaster should lie mainly with the actuaries.
- Professions have a responsibility to learn about advances in relevant disciplines.