

# Partial Delaunay triangulation and Bluetooth scatternet formation

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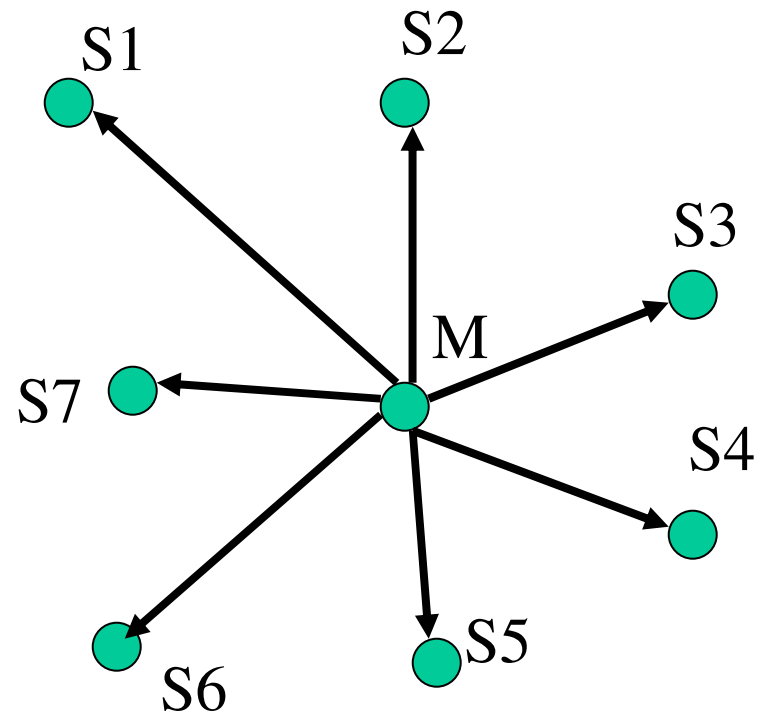
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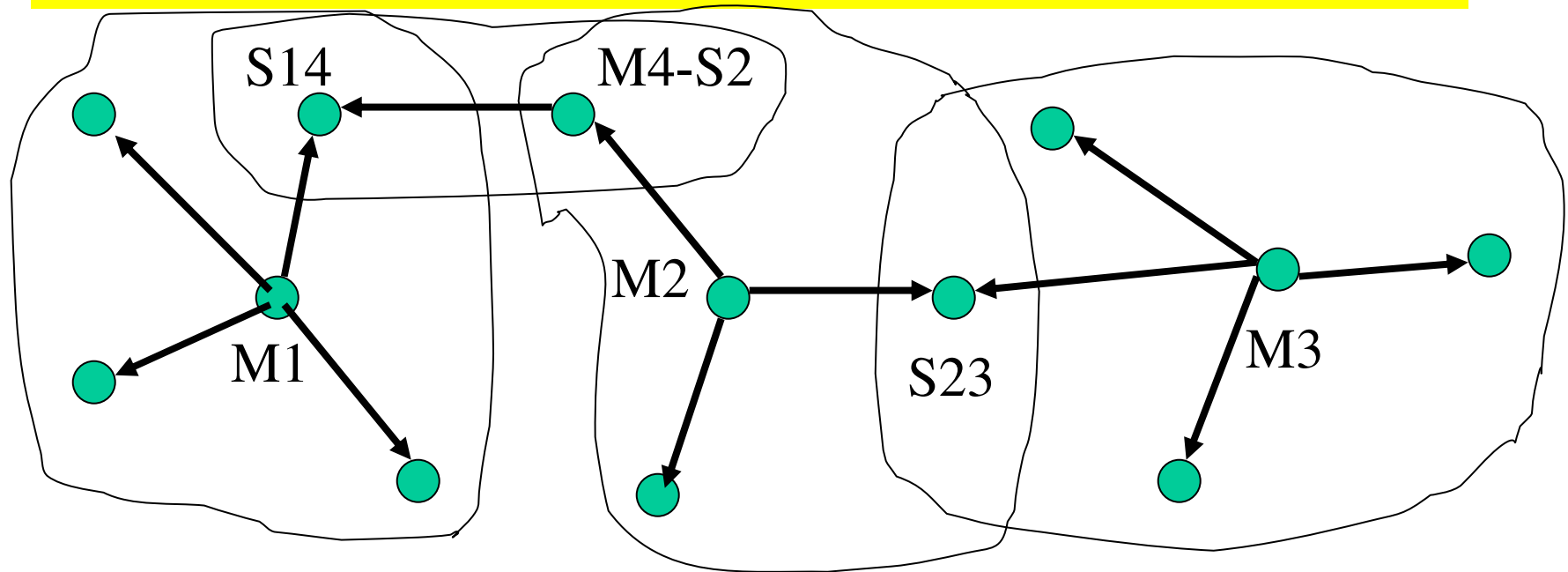
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# Bluetooth - piconet

- Short-range
- Master-slave
- Frequency hopping
- PICONET = master +  
K slaves,  $K \leq 7$
- Additional slaves must  
be parked



# Bluetooth - scatternet



Connect piconets into scatternet

Avoid master-slave bridges

Bridges participate in piconets on time division basis

Minimize number of slave roles

# Scatternet by growing tree

- Ramachandran, Kapoor, Sarkar, Aggarwal 2000:  
grow tree from root, master not always directly connected to its slave
- Zaruba, Basagni, Chlamtac 2001:  
grow tree from root, at most 5 slaves per master;  
if  $>5$ , select two connected slaves, link them, and disconnect one;  
Multiple blueroots extension
- *Communication overhead and Scatternet maintenance?*
- Salonidis, Bhagwat, Tassiulas, LaMaire 2001:  
centralized, max 36 nodes
- Law, Mehta, Siu 2001:  
single-hop networks (complete graph)

# Clustering based scatternet formation

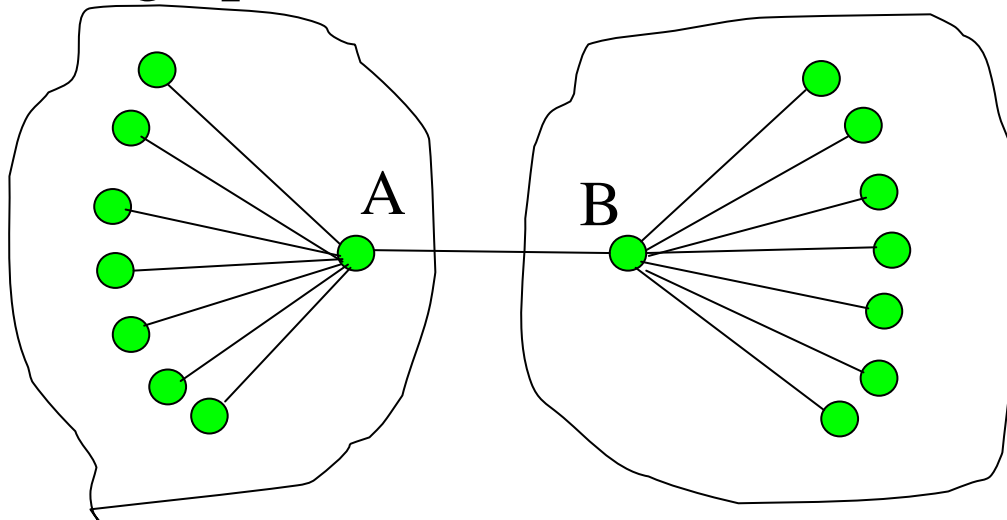
- Basagni, Chlamtac, Petrioli 2001
- Detect neighboring nodes by paging and scanning
- Apply clustering process
- Clusterheads = masters
- Nodes in a cluster = slaves
- Connect clusters = bridge piconets
- *degree (number of slaves) not limited to 7*
- parking and unparking process ?
- Maintenance is not localized – chain effect ?

# Neighbor discovery in Bluetooth

- Each node decides with probability 0.5 between inquiry and inquiry-scan modes
- Senders and receivers change frequencies in mutually random pattern ( ? 32 frequencies )
- If sender and receiver are on the same frequency at some time, they discover each other, and establish master-slave relation
- In multi-hop networks, overall connectivity established quickly, but full awareness of all neighbors is slow

# Scatternet by random key clustering

- Wang, Thomas, Haas 2002
- Guerin, Kim, Sarkar 2002
- Node decides to be master at random
- and then 'slaves' up to seven neighboring nodes
- Connect scatternet by bridge piconets
- No bridge piconet, disconnected scatternet ?

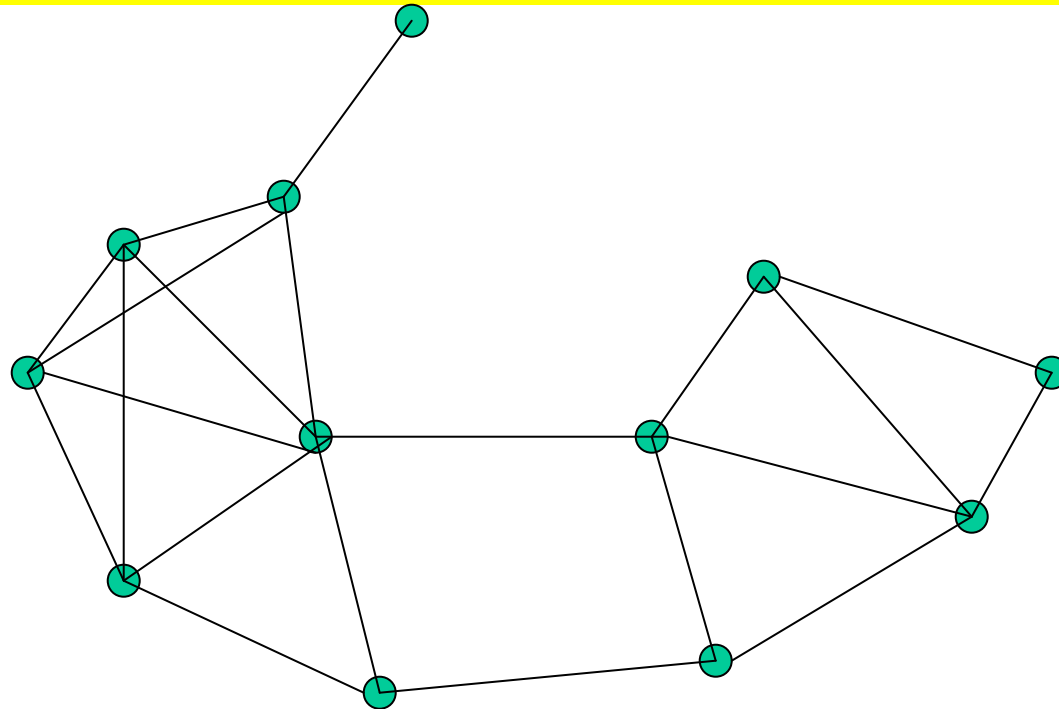


# Degree limited connected scatternet formation

- Li, Stojmenovic 2001 – clustering based
- Stojmenovic 2002 – dominating set based
- *Phase I* = create unit graph and construct a planar connected structure in localized manner
- *Phase II* = eliminate some edges in the planar structure to limit the degree of each node to 7
- *Phase III* = decide master-slave roles between two nodes of each edge in the structure
- Only phase III differs in clustering vs. dominating set based formation; planar structure is optional



# Scatternet formation – phase I



Unit graphs  
radius

Create unit graph

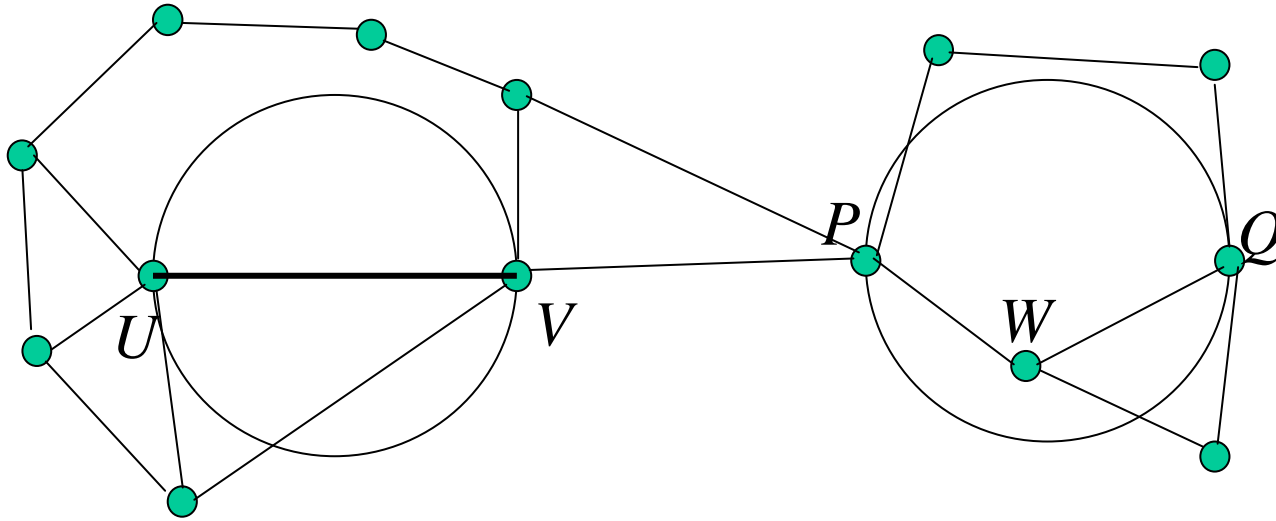
Assumption: Each node is aware of its position and learns position of all neighbors within transmission radius

Construct planar structure in localized manner:

Gabriel graph **GG**, Relative neighborhood graph **RNG**,

Partial Delaunay triangulation **PDT**

# Gabriel graph



Gabriel graph  $GG(S)$  contains an edge  $(U, V)$   
iff the disk with diameter  $(U, V)$   
contains no other point from  $S$

Computing  $GG$  from unit graph requires no message exchange

# Gabriel graph properties

**Planar** – no two edges intersect

**Connected** - Contains MST (minimal spanning trees)

Unit graph contains MST

Planar graph with  $n$  nodes has at most  $3n-6$  edges

Average degree of a planar graph is  $< 6$

RNG has average degree  $< 2.4$  = too sparse

RNG is subset of GG

*Intersection of GG and unit graph is connected and planar*

# Partial Delaunay Triangulation

Li, Stojmenovic 2001  $\text{RNG} \subseteq \text{GG} \subseteq \text{PDT} \subseteq \text{DT}$

Delaunay Traingulation = dual Voronoi diagram

UV in DT iff there exist a circle with chord UV without other nodes inside it

Test disk with diameter UV:

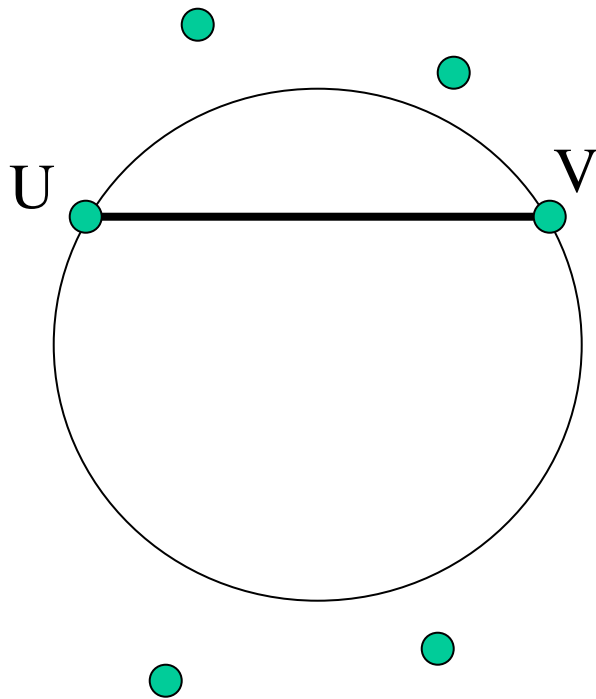
If empty then UV in PDT

If nodes inside disk on both sides then not in PDT

Find smallest angle on both sides of UV

If together  $\geq \pi$  then not in PDT

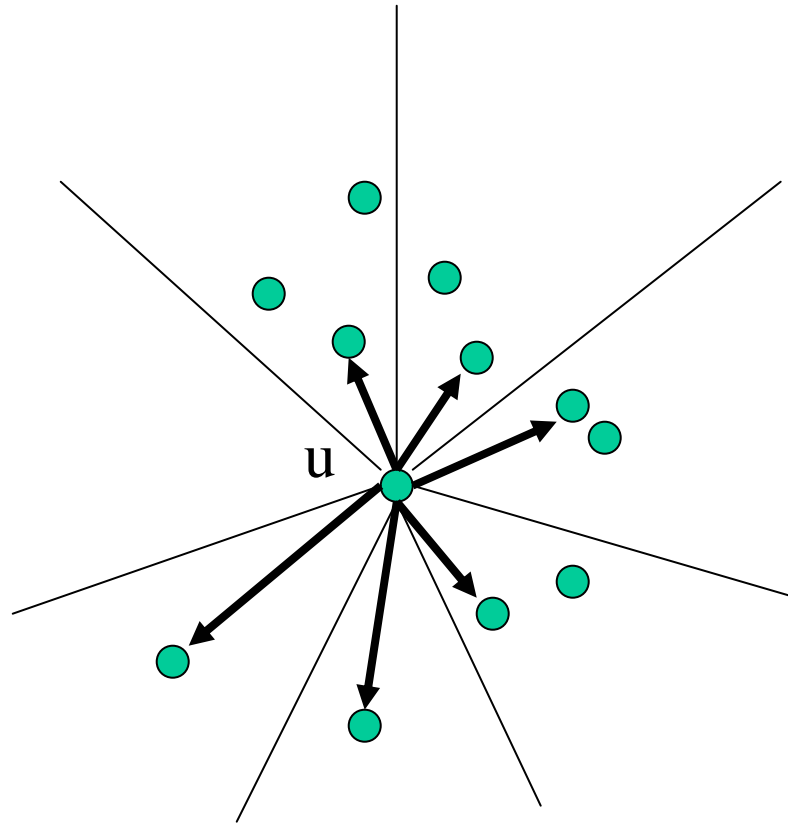
If together  $< \pi$  then in PDT iff both are neighbors, using 1-hop or 2-hop info



PDT= portion of DT which can be decided locally

# Yao graph

$k=7$

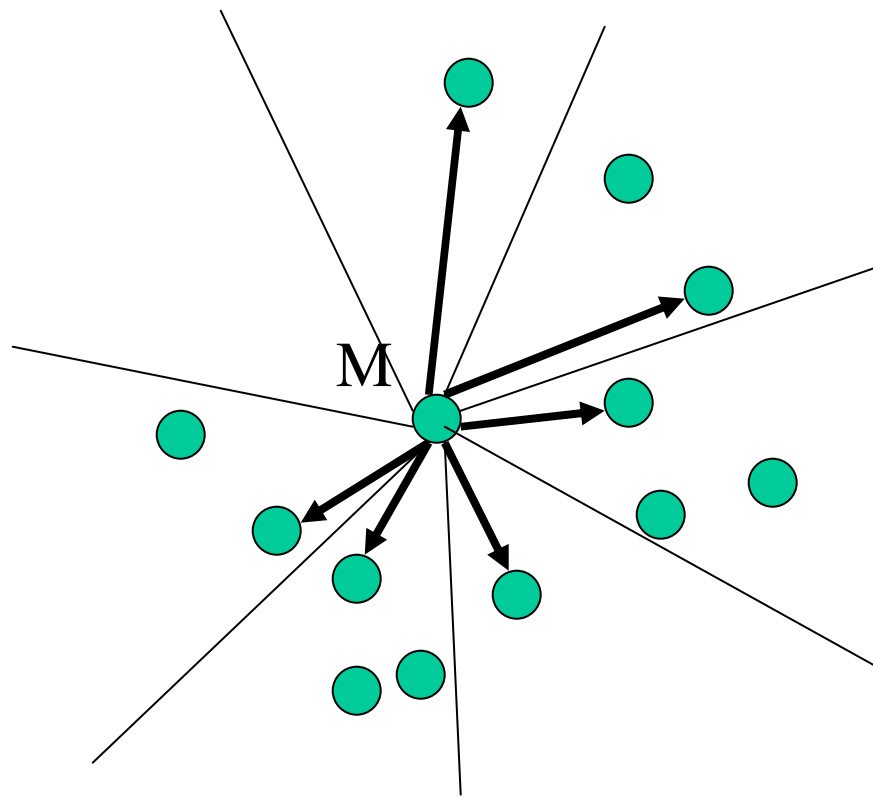


Divide into  $k$  equal cones around  $u$

Find closest point in each cone, if any

# Limiting degrees – cluster based

**Phase 2:** Applied on **active** nodes = nodes with highest keys among undecided neighbors



- apply Yao construct

- delete other edges

- undirectional graph remains

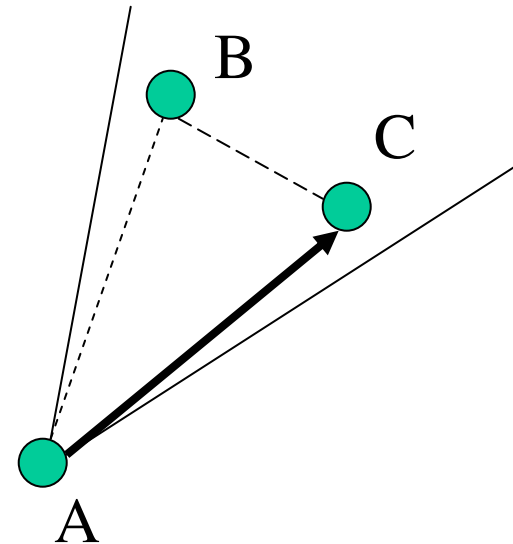
# Deciding master-slave roles

- Key= Bluetooth ID (one-hop neighbor discovery)
- Key= (degree, ID) (two-hop neighbor information needed)
- Active node decides on roles on each undeleted edge:
- Higher original key, or
- Clustering based: clusterhead= master, border node slave and master role given to other neighbor, which can be clusterhead or the second node for two-node gateway piconet

# Yao construct preserves connectivity

- Sort all edges of unit, GG or RNG by key=(length, survive), survive=0 if bi-directional in Yao, =1 if not
- Construct MST by considering edges in increasing order, include if no cycle created

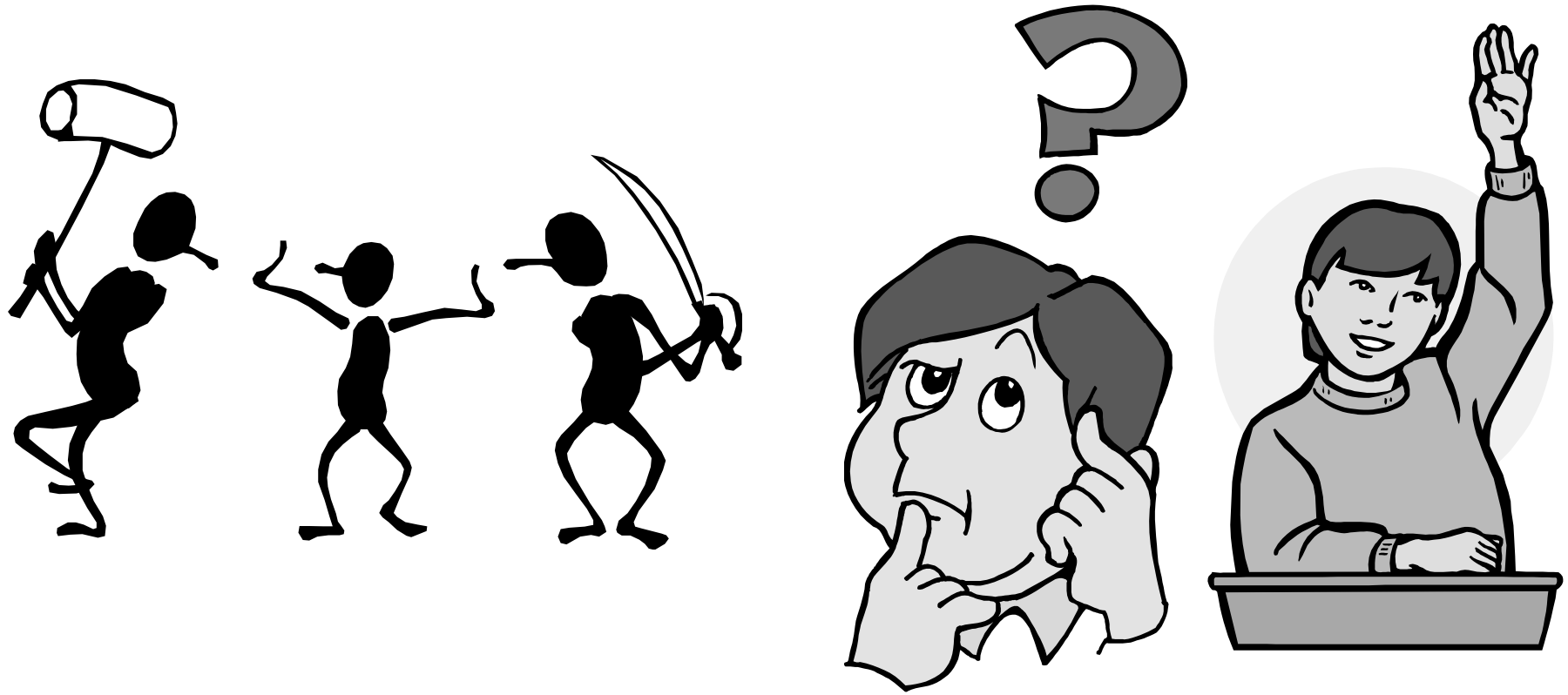
If AB and BA not in Yao then  $\exists C$ ,  $AB \leq AC$ ,  $BC < AB \Rightarrow BC$ , AC already considered for MST and connected, AB not needed in MST  
If AB is directional then  $\exists$  path between them consisting of shorter edges and an edge of same length but bi-directional (proof involved)  $\Rightarrow$  AB not needed in MST





# Future work

- Experiments
- Bluetooth scatternet formation without position information
- Routing in scatternets
- Power efficient scatternets
- Denser planar graphs ?
- Neighbor discovery and non-unit graphs
- Scheduling, capacity, ...
- Three-dimensional scatternets



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