

Computational Techniques for MtM and Risk Management of Loan Portfolios

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Dan Rosen
VP Research & New Solutions

Algorithmics Inc.

drosen@algorithmics.com



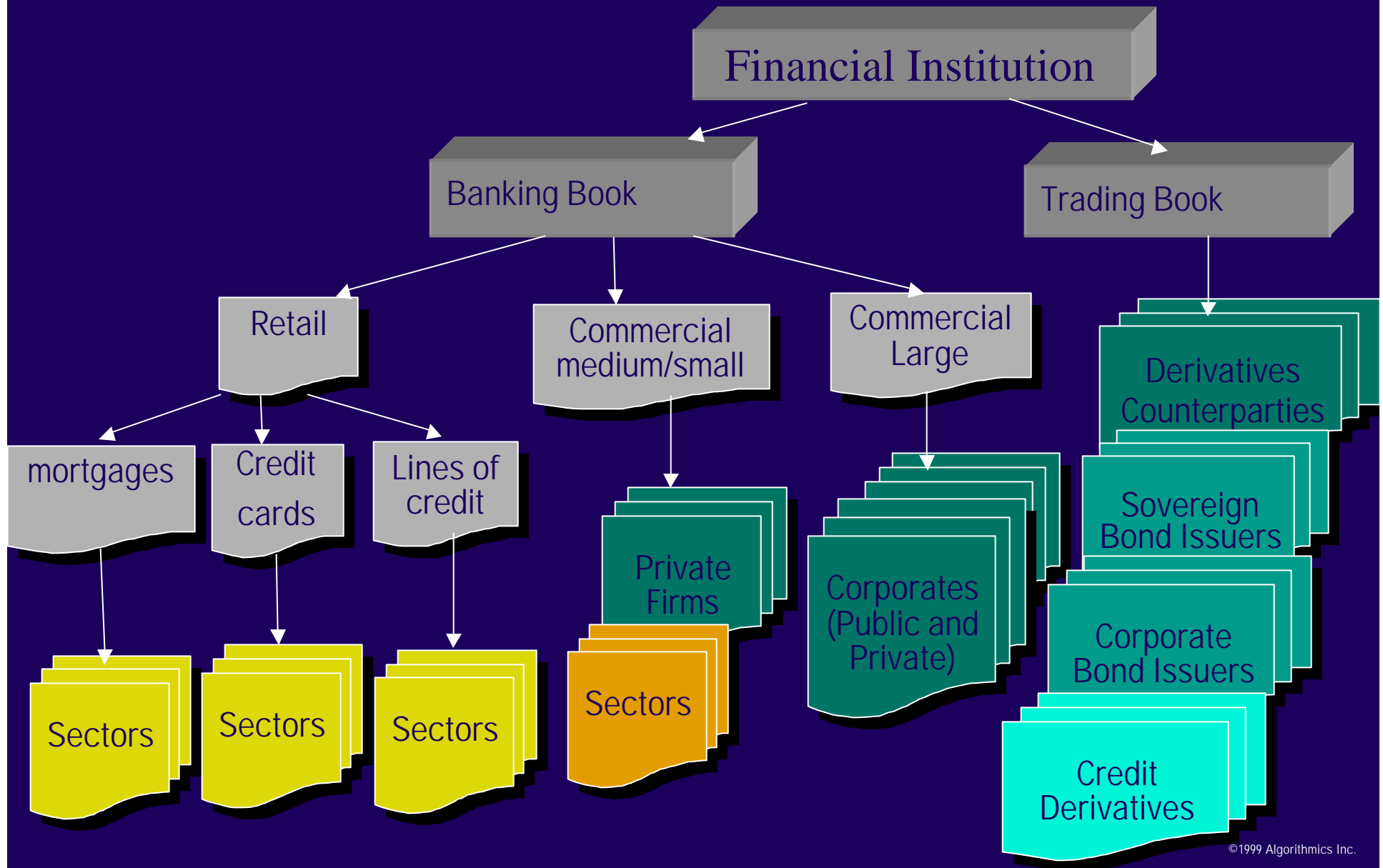
Outline

- Enterprise credit risk management & valuation
- Loan Valuation and MtM
 - MtM of Loans
 - properties & embedded options
 - underlying credit model
- Loan valuation Framework
- Calibration in practice
- Examples

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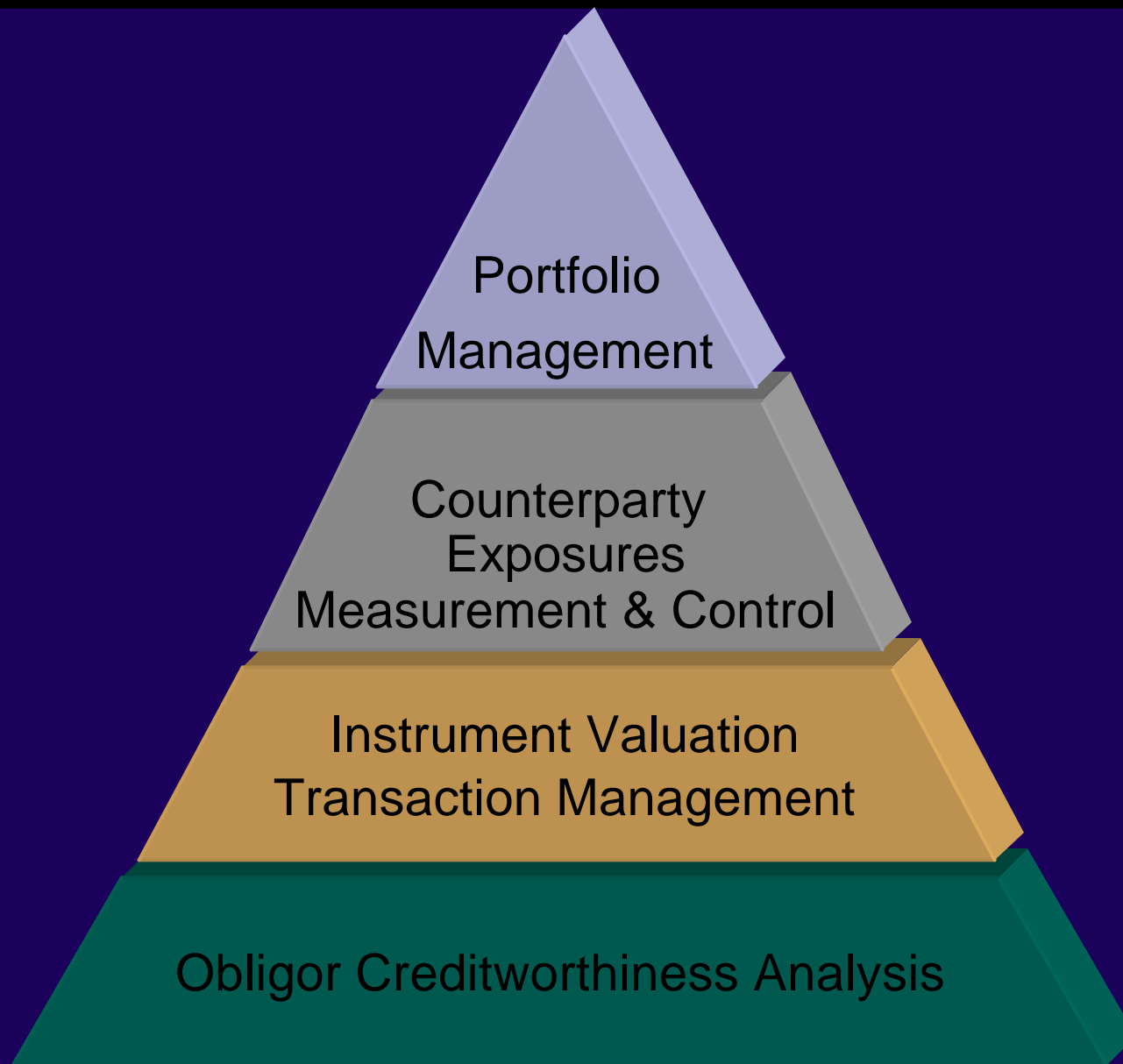
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Enterprise Credit Risk



Enterprise Credit Risk Functions

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Credit Risk Pricing

– Some myths

Myth #1. In practice, we cannot apply no-arbitrage models in credit

- standard model assumptions are not met
- cannot hedge credit as well as market risk; no liquidity, etc
- No-arbitrage pricing is the basis managing risks (trading book):
 - price and hedge securities, MtM portfolios and measure risk
 - still... models used in practice make assumptions that are not met!
- In practice, “no-arbitrage” models lead to powerful insights:
 - systemic way to compare prices, understand/strip structure, hedge
- Wealth of credit risk models available

“No-arbitrage” pricing:

- Model of underlying credit (and market) processes
- Calibration: extract the basic prices from the market
 - simpler, “liquid” risk-comparable securities
- Model accurately the **structure and cashflows** of credit instruments
- Output: prices (and sensitivities, etc.) of more complex credit securities

Credit Risk Pricing: banking loans

Myth #2. We cannot apply no-arbitrage models in credit

- MtM approach to loans is an academic exercise - useless in practice
- In practice, management of loan books is mostly simplistic and static
- Most prevalent method for pricing and managing loans based on RAROC
- Application of option valuation to bank loans has been much slower
 - credit risk modeling is complex ---> has trailed behind market risk
 - shared “pessimistic” view on applicability of no-arbitrage to credit risk
 - standard practice of static management of (illiquid) credit risks.
- Reality: Credit risk pricing and active management are now here to stay...

Why is Pricing & Valuation Important ?

Evolution toward “efficient” portfolio investor approach:

- Move away from “originate & hold” business model
- Separation of origination & portfolio management (P/L)
- Credit risk transfer pricing is required
- Assessing hedge effectiveness → evaluation of instrument structure

Need to move beyond RAROC pricing:

- Calibration focus has tended to be “internal” not “market-based”
- Doesn’t reflect loan structure & embedded optionality
- Not a no-arbitrage approach

Why is Pricing & Valuation Important ?

Substantial arbitrage opportunities exist today

- Complicated loan structures interact in a non-transparent way
 - prepayment, utilization, grid pricing, term out options etc.

Portfolio Credit Risk generally based on simplistic valuation approaches

- General treatment of loans as if they were “simple” bonds
 - over-estimation of credit capital (everything else the same)
 - Complicated loan structures tend to mitigate value volatility in loans relative to bonds
- Must understand of embedded options & market-credit interaction

“Mark to Market” calibration has become a reality for credit instruments

Credit Risk Pricing: banking loans

Myth #3. No-arbitrage models in credit - so what's the big deal anyway?

- Wholesale bank loans, corporate bonds and credit derivatives:
more than \$30 trillion USD in exposures worldwide!
- Enormous potential business benefits from effective valuation & risk-management (understanding effect of structure and optionality on value)
 - better **pricing** and **structuring** of credit risky instruments
 - more flexible and **dynamic management** of credit portfolios
 - greater exploitation of **arbitrage** opportunities
 - more accurate **portfolio credit risk** modeling

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Credit Risk Pricing: banking loans

Myth #4. Loan structure is less important - get right the ratings, PDs, etc.

- Fact: great advances in understanding credit and quantifying an obligor's ability to meet an obligation
 - obligor's default behaviour has a major effect in valuation and risk
- ... however, other properties of the facilities ALSO have a major effect:
 - embedded options and schedules
 - collateral: value, volatility and correlation to underlying
- Fact: loans are not straight bonds!
 - embedded options play an important, yet mostly neglected, role

Myth #4. Structure is less important

Example: syndicated deal (14/04/00): \$115 M to fund acquisition of PlayCore Holdings Inc. (unrated holding company: interests in sporting and games)

- \$30 million revolver, \$25 million term loan A, \$60 million term loan B.
- Secured credit: 85% of eligible accounts receivable, 60% of eligible inventories, plus \$3,000 monthly from November through March
- Covenants require hedging of IR risk, minimum fixed-charge coverage ratios, limitations on dividends, etc.
- Pricing tied to: Funded debt / EBITDA
- In default, pricing increases by 200 bps
- Prepayment without penalty at any repricing date.

Credit Risk Pricing: banking loans

Myth #4. Structure is less important... Example of large corporate loan

Term-loan B component (marketed to loan funds):

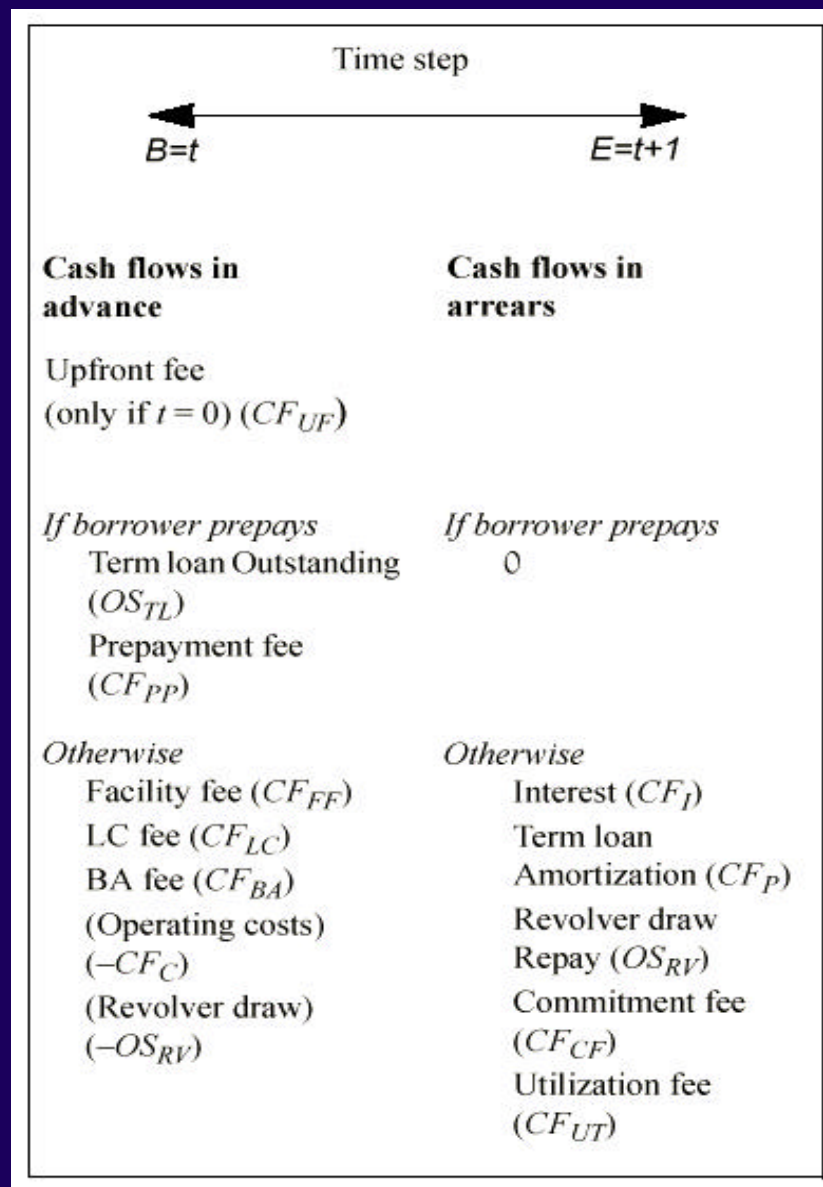
- Maturity July 1, 2006 (87 months term)
- 20 quarterly payments of \$150,000, starting on October 1, 2000
- Followed by eight quarterly payments of \$7,125
- Loan amortization over several quarters
- Initially, facility priced at
PRIME + 225bps (LIBOR + 400bps)
- Pricing grid determines pricing

Level	Debt to cash flow ratio	Prime + (bps)	LIBOR + (bps)
1	4.75 or greater	250	425
2	[4.25, 4.75)	225	400
3	less than 4.25	200	375

Table 1: Pricing grid of PlayCore term loan B
(LPC Gold Sheets 2000a)

Modelling a Bank Credit Facility

- Choice of credit from among a set of instrument types:
 - a term loan
 - a funded revolving line
 - a letter of credit
 - banker's acceptance.
- Vital to model cash flows accurately



Modelling a Bank Credit Facility

$$ECF = (CF_{UF} + OS_{TL} + CF_{PP}) \cdot P_P$$

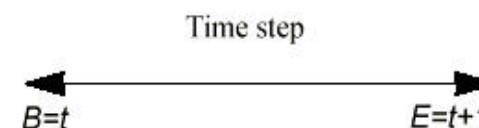
$$+ [(CF_{UF} + CF_{FF} + CF_{LC} + CF_{BA} - OS_{RV} - CF_C)$$

$$+ (1 + R)^{-1} \{ (1 - P_D)(CF_I + CF_{CF} + CF_{UT} + CF_P + OS_{RV})$$

$$+ P_D(1 - L)(CF_I + CF_{CF} + CF_{UT} + OS_{TL} + OS_{RV})$$

$$- P_D \cdot L \cdot (AC \cdot (REU + (1 - REU) \cdot LEQAC) - OS_{TL} - OS_{RV}) \}]$$

$$\times (1 - P_P)$$



Cash flows in advance

Upfront fee
(only if $t = 0$) (CF_{UF})

If borrower prepays
Term loan Outstanding
(OS_{TL})
Prepayment fee
(CF_{PP})

Otherwise
Facility fee (CF_{FF})
LC fee (CF_{LC})
BA fee (CF_{BA})
(Operating costs)
($-CF_C$)
(Revolver draw)
($-OS_{RV}$)

Cash flows in arrears

If borrower prepays
0

Otherwise
Interest (CF_I)
Term loan
Amortization (CF_P)
Revolver draw
Repay (OS_{RV})
Commitment fee
(CF_{CF})
Utilization fee
(CF_{UT})

Modeling Embedded Options

- **Default option**: in default, borrower may not pay an obligation in full
 - affects CFs explicitly through the probability of default
- **Prepayment option**: right to prepay or cancel the contract before maturity
 - affects CFs explicitly through the probability of prepayment
 - function of obligor credit state, risk-free interest rates and spreads
 - contingent on credit events other than default (e.g. credit downgrades)
- **Credit line utilization option**: right to choose the usage level of a commitment
 - affects implicitly several CFs and outstanding amounts - as obligor's creditworthiness diminishes, draw on credit line increases
 - embedded option on credit events other than default (e.g. downgrades)

Analyzing Complexities in Credit Agreements

-- Need Much More than Two-State Approach

State Change	Cash Flow Effect	Modeled by:
Creditworthiness improves without pricing change	Borrower prepays all of the outstanding loan principal and seeks new financing at lower spreads	Multiple credit states
Creditworthiness drops without pricing change	Borrower draws down the credit line, creating more interest payments but greater exposure with higher risk	Multiple credit states
Deterioration in rating or financial ratio leads to pricing step-up	Spread and fee rates rise, producing higher payments and curtailing the borrower's incentive to draw more credit	Multiple credit states
Deterioration in rating or financial ratio triggers covenant violation	Creditor drops the line or demands better collateral coverage, reducing potential default losses	Multiple credit states
Interest rates fall	Borrower with callable, fixed-rate obligation prepays all of the outstanding principal and seeks new financing at lower rates	Interest rate factor
Interest rates rise	Borrower with interest rate cap in a floating-rate agreement owes less than otherwise	Interest rate factor
Credit spreads for all risk grades decrease	Borrower prepays all of the outstanding loan principal and seeks new financing at lower spreads	Credit spread risk factor

The 2-state (default/non-default) credit model misses many of these cash flow contingencies.

Large Corporate Example: \$10 Million Primary Participation in Playcore

Playcore 7-Year Term Loan B Tranche: B- Counterparty

	NPV	Duration*
• Base Case Valuation	-\$267k	2.31 years
• No Prepayment:	-\$126k	4.87 years
• Prepayment Option	\$141k	
• No Pricing Grid	-\$270k	
• No Amortization (NPV)	-\$286k	
• Key point: substantial impact on value of loan structure components		
(NOTE* Duration is risk and option-adjusted)		

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Underlying Credit Risk Model

In summary:

- The cash flows from credit facilities are a function of: borrower **creditworthiness** (e.g., risk rating), **interest rates** and credit **spreads**.
 - e.g. a decrease in interest rates or credit spreads or an improvement in borrower risk rating may trigger prepayment
 - credit facilities include pricing grids, graduated utilization fees and amortization schedules
- Underlying credit risk model must describe each state of the world by
 - obligor **creditworthiness** (e.g. a ratings and default probabilities)
 - the term structure of **default-free interest rates**
 - the term structures of credit **spreads** for non-defaulted securities.

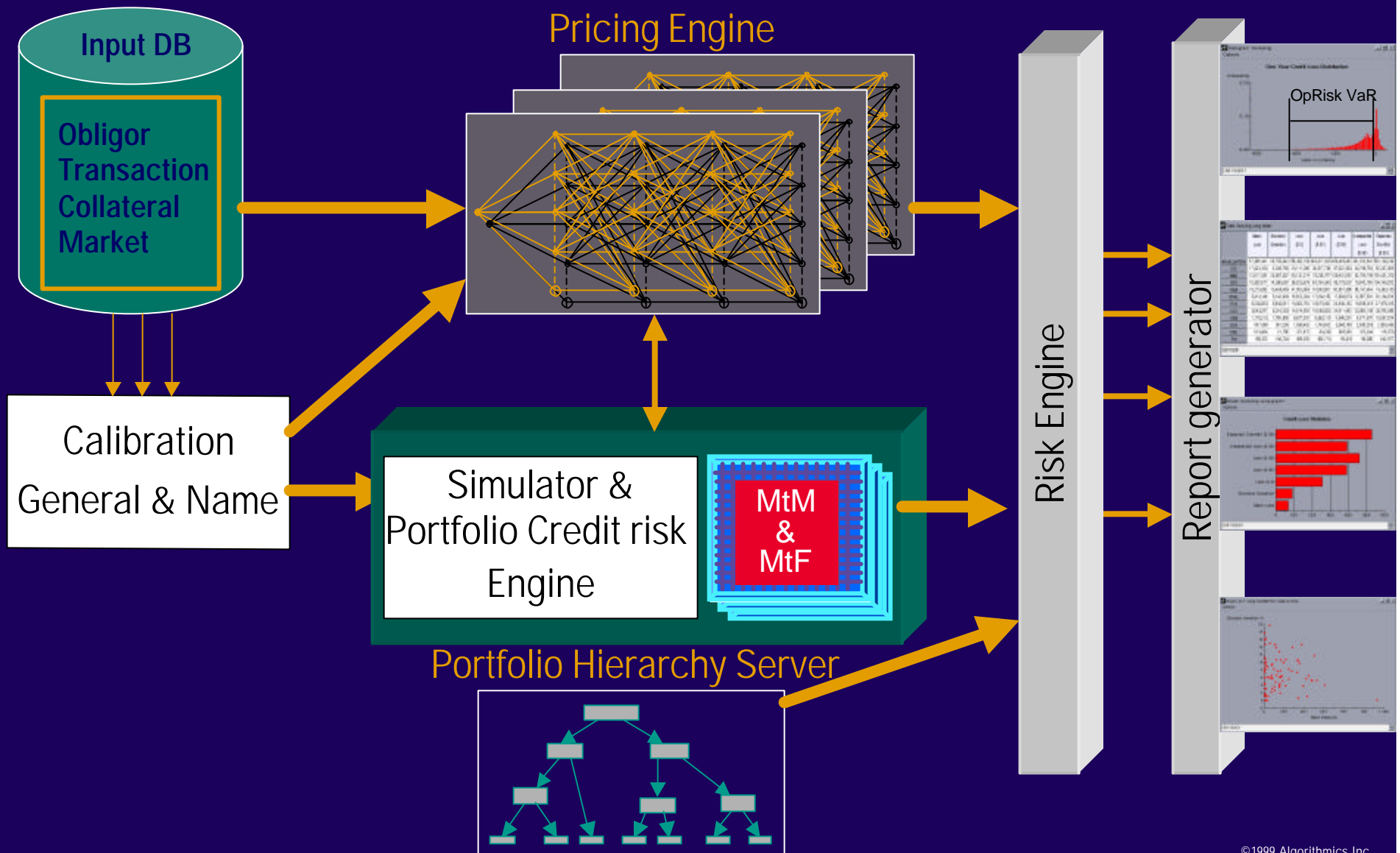
Underlying Credit Risk Model

- Multi-credit state (rating-based) models particularly suitable
 - e.g. Jarrow, Lando and Turnbull 1997, Lando 1998; loan applications in Ginzberg et al. 1994, Aguais et al. 1998, Aguais & Santomero 1998
- Some theoretical & practical challenges (from high dimensionality)
 - require structure to reduce the dimensionality (JLT, Lando, Kishima-Kobayashi, Aguais et al)
 - calibration: start with real transition matrix (e.g. S&P or Moody's), then apply a low-dimensional process - modify transition matrix to fit to observed term structure of market spreads (risk-neutral measure)

Outline

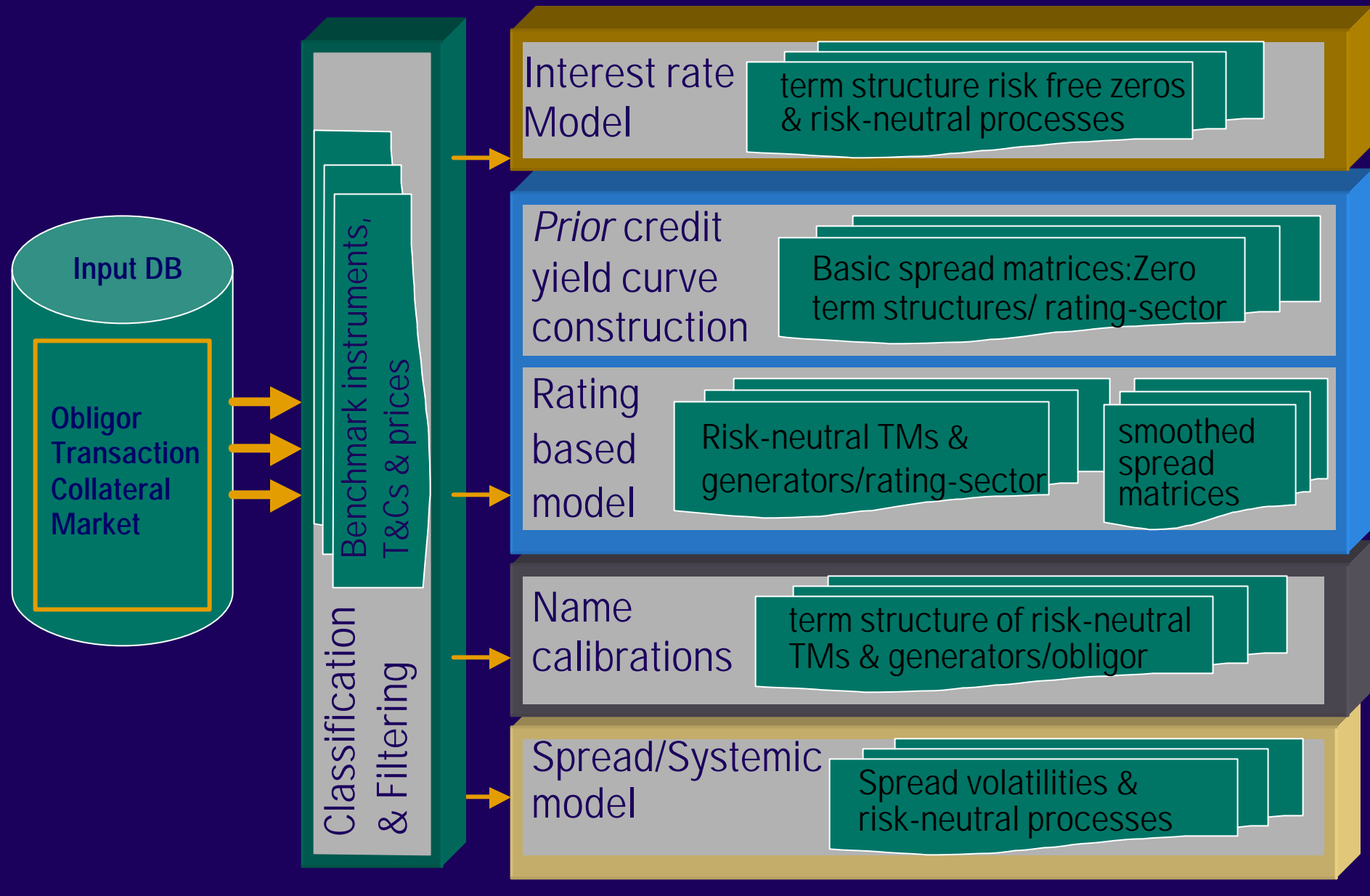
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Loan MtM & Risk Engine: functional architecture

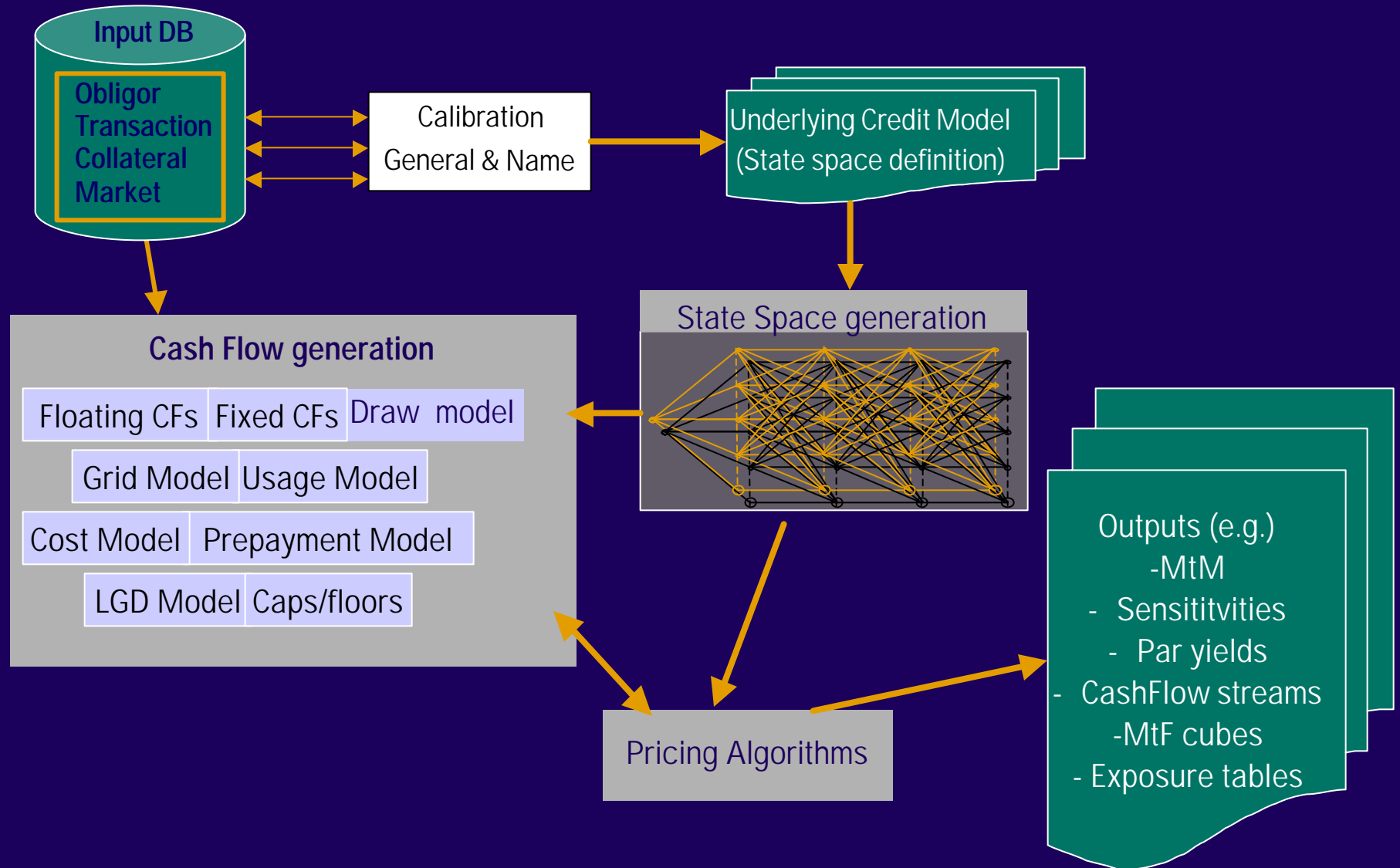


Credit Calibration Engine: Functional Architecture

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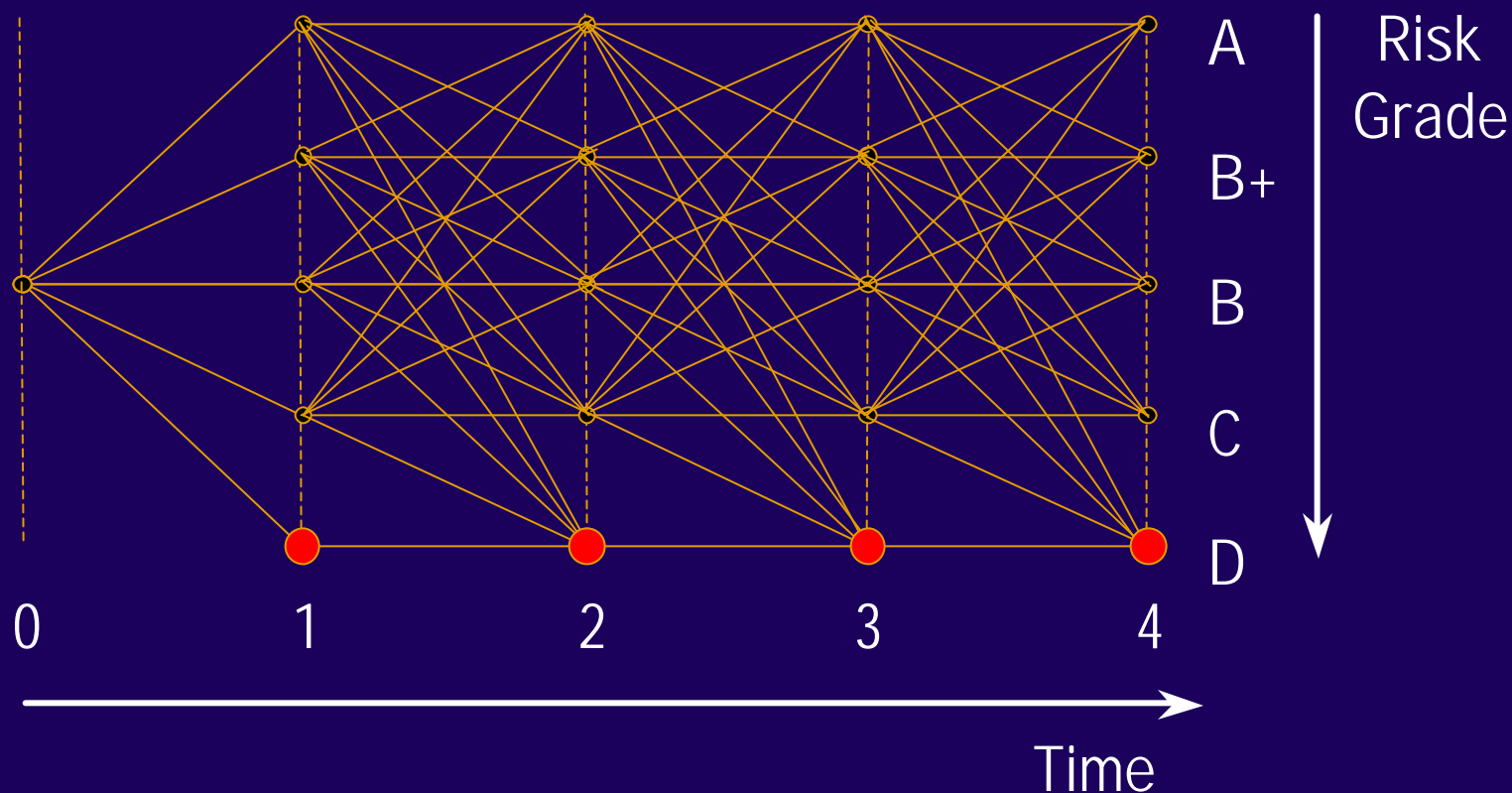
ACV Pricing Engine: Flows (Approx.)



Lattice Methods for MtM

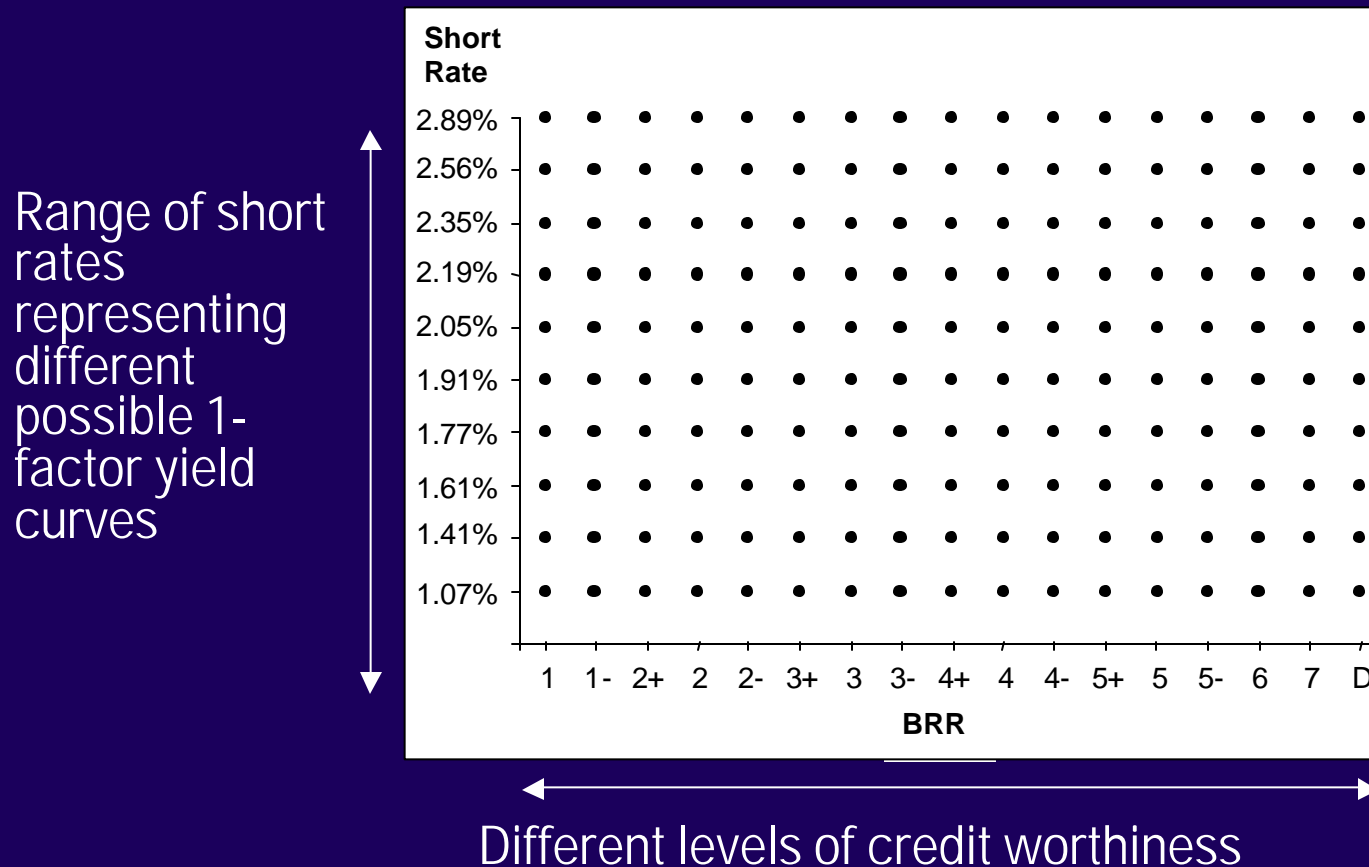
Calculating Expected NPV:

- Primary credit factor is the borrower creditworthiness
- Options-exercise decisions take place at each node
- valuation using backwards recursion through the grid



Example: Lattice With Credit & Interest Rate Factors

At each time point, the ACV lattice depicts all possible levels of credit worthiness crossed with all levels of interest-rates



Cashflow Generation: Prepayment Option

- Rational borrower exercises option to prepay if the market value of the loan, conditional on continuing, rises high enough above par to pay for:
 - any prepayment penalty
 - refinancing transactions costs of the borrower
 - origination costs (for an efficient lender)
- Perfect decision (PP= 0 or 1) : borrower prepays if savings in switching to a new loan relative to the existing above-par loan more than cover transactions costs
- Imperfect decision: PP as a continuous monotonic function of the predicted prepayment savings (more realistic but difficult to obtain data to calibrate this function to actual borrower behavior)
- Require both the lender's and the market's costs (of competitive providers of credit) of originating and of servicing loans
- Borrower costs of transacting a new loan must also be determined

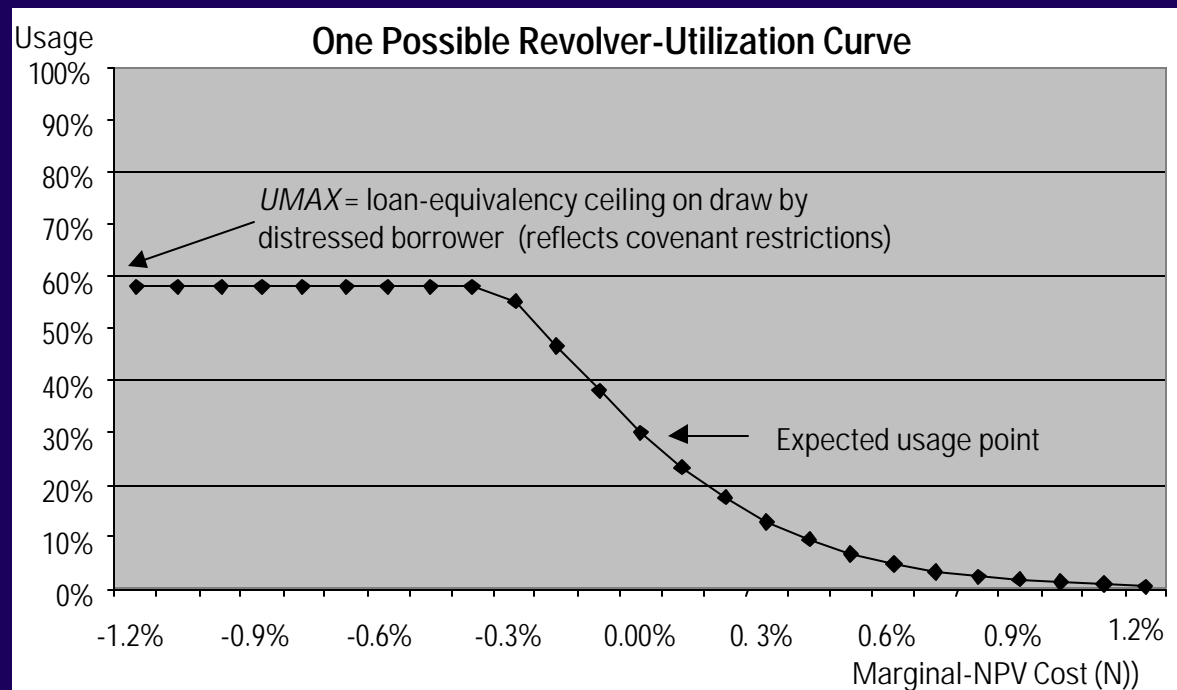
CashFlow generation: Credit Line Utilization

- Borrower's option to choose the usage of the line
- The usage of a line influences
 - the payments that the borrower owes to the creditor
 - the amount of exposure that the creditor bears
- In the equation giving expected CFs, it affects several cash flows and outstanding amounts
 - Any remaining commitment above term loan is available to the borrower, assuming compliance with the loan covenants: borrower may use this amount in varying degrees from 0% to 100%.
- The usage model determines two components:
 - the overall usage, RUACA, of the available commitment
 - the relative usage of the different instrument options: the funded revolver, the letter of credit and the banker's acceptance.

Credit Line Utilization cont.

Overall usage of the available commitment $RUACA = f(\text{net credit line cost})$

- rises above its anticipated value if marginal cost of drawing credit becomes cheap (low relative to the market par cost of obtaining credit)
- falls if the marginal cost becomes expensive (high relative to the market par cost of obtaining credit)



Pricing Engine architecture

Some characteristics of the Pricing Engine and the outputs

- Parallel processing: applications can call as many PEs as available which dump results in MtF database
- Extensible and flexible: reusable libraries at each level
- Applications:
 - Portfolio Loan MtM analysis: PE results passed directly to risk engine for portfolio analysis
 - Portfolio credit risk and capital: PE results are inputs to simulation and PCR engine (efficient computational schema required)
 - Front office: loan pricing & structuring; marginal capital limits; transfer pricing; what if analysis.

Expected Cashflows - PlayCore Term Loan - Risk Adjusted

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ACV: Cash Flow Report - Microsoft Internet Explorer

Address: http://localhost:9090/control/cashFlow.jsp

[Return to Summary](#)

Period	Discounted Total	Total	Interest	Fees	Principal	Costs	Cumulative Prepayment	Cumulative Default
1	-9,108,842	-9,108,842	231,845	12,500	-9,308,874	-44,314	0.00	2.83
2	715,250	725,001	211,139	0	522,086	-8,223	5.05	5.47
3	892,431	916,819	195,689	0	728,811	-7,681	8.29	7.90
4	848,985	883,827	176,336	0	714,329	-6,838	13.81	10.10
5	596,968	629,522	156,316	0	479,440	-6,233	19.38	12.75
6	552,252	589,852	141,159	0	454,497	-5,804	22.27	15.22
7	540,028	584,051	127,131	0	462,253	-5,333	25.06	17.49
8	433,189	474,204	113,575	0	365,385	-4,755	28.08	19.53
9	378,845	419,485	104,836	0	319,077	-4,428	30.29	21.62
10	365,607	409,582	99,425	0	314,289	-4,132	32.01	23.56
11	379,283	430,227	93,781	0	340,242	-3,796	33.81	25.35
12	292,360	336,001	87,199	0	252,170	-3,368	36.00	26.96
13	264,930	308,597	80,307	0	231,425	-3,134	37.41	28.54
14	258,266	304,973	72,159	0	235,728	-2,914	38.65	30.01
15	233,315	279,193	64,295	0	217,557	-2,659	40.01	31.37
16	199,767	242,116	57,221	0	187,288	-2,393	41.29	32.59
17	180,126	220,960	52,425	0	170,725	-2,190	42.34	33.77
18	164,918	204,771	49,257	0	157,540	-2,026	43.26	34.87
19	149,658	188,237	46,214	0	143,875	-1,853	44.11	35.87
20	133,940	170,764	43,330	0	129,095	-1,660	44.88	36.78
21	121,577	157,174	39,638	0	119,072	-1,536	45.58	37.65
22	238,775	313,071	35,321	0	279,170	-1,419	46.21	38.46
23	202,789	269,548	27,560	0	243,127	-1,138	46.78	39.20
24	171,140	230,476	21,066	0	210,284	-875	47.31	39.87
25	145,465	198,309	16,114	0	182,873	-678	47.78	40.50
26	122,991	169,771	12,164	0	158,111	-504	48.20	41.09
27	99,865	139,700	8,588	0	131,460	-348	48.59	41.63
28	158,269	224,550	5,480	0	219,284	-214	48.76	42.11
Total:	-267,852							

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Expected Cashflows - PlayCore Term Loan -- Non-Risk Adjusted

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ACV: Cash Flow Report - Microsoft Internet Explorer

Address: <http://localhost:9090/control/cashFlow.jsp>

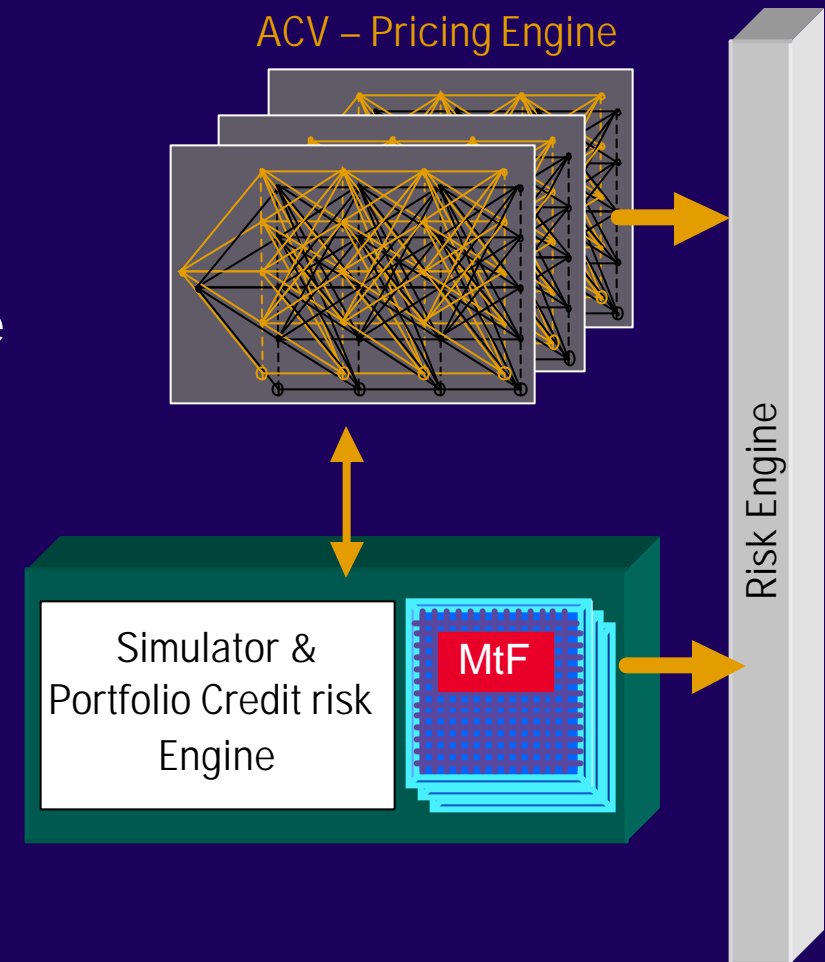
EXPECTED CASHFLOWS								
Period	Discounted Total	Total	Interest	Fees	Principal	Costs	Cumulative Prepayment	Cumulative Default
1	-9,034,088	-9,034,088	232,357	12,500	-9,234,631	-44,314	0.00	2.13
2	752,524	762,783	210,382	0	560,566	-8,165	6.29	4.09
3	968,057	994,511	194,359	0	807,749	-7,597	10.39	5.90
4	918,563	956,260	173,571	0	789,388	-6,699	17.14	7.51
5	657,906	693,783	152,665	0	547,158	-6,039	23.89	9.00
6	607,210	648,551	136,732	0	517,389	-5,570	28.28	10.38
7	600,613	649,576	122,029	0	532,611	-5,064	32.47	11.63
8	462,839	506,661	107,622	0	403,494	-4,454	36.93	12.75
9	399,860	442,754	98,697	0	348,168	-4,111	40.18	13.78
10	390,755	437,754	93,103	0	348,462	-3,810	42.94	14.73
11	419,481	475,823	87,121	0	392,170	-3,468	45.78	15.60
12	301,614	346,636	79,731	0	269,933	-3,027	49.15	16.37
13	271,797	316,597	73,010	0	246,383	-2,796	51.34	17.09
14	272,044	321,242	65,224	0	258,600	-2,582	53.34	17.75
15	243,810	291,751	57,501	0	236,580	-2,329	55.52	18.35
16	203,017	246,054	50,610	0	197,516	-2,072	57.52	18.88
17	181,999	223,258	46,032	0	179,106	-1,880	59.16	19.37
18	165,185	205,103	42,940	0	163,888	-1,725	60.65	19.83
19	148,568	186,866	39,997	0	148,433	-1,564	62.02	20.24
20	131,722	167,936	37,231	0	132,096	-1,391	63.27	20.60
21	118,845	153,643	33,846	0	121,074	-1,277	64.38	20.94
22	215,131	282,069	29,964	0	253,277	-1,171	65.40	21.25
23	180,667	240,143	23,228	0	217,848	-933	66.34	21.54
24	150,564	202,765	17,638	0	185,839	-712	67.19	21.79
25	126,515	172,474	13,419	0	159,604	-548	67.95	22.02
26	105,655	145,841	10,078	0	136,168	-405	68.64	22.23
27	82,658	115,630	7,078	0	108,830	-278	69.28	22.43
28	132,656	188,210	4,563	0	183,820	-172	69.58	22.60
Total:	176,164							

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Simulation engine, PCR and MtF

- Valuation is costly; we require ingenious algorithms to do simulations for stress testing and statistical risk measurement
- Pricing engine must be leveraged to devise fast computational algorithms
- The choice of a multi-state credit pricing infrastructure is particularly powerful and consistent with Portfolio measurement
- In addition to MtM of the loans, intermediate results and other calculated parameters can be used to speed-up simulations



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ACV Calibration Approach Has Two Major Components

Baseline Calibrations: Develop one or more baselines using one of the large databases that provides estimated market prices for thousands of credit instruments spanning many maturities & levels of credit worthiness. Use these calibrations in pricing when more detailed data are lacking.

Name or Sector Calibrations: Adjust the appropriate baseline to get better accuracy for companies or sectors with credible, name- or sector-specific credit curves.

Baseline Calibration Involves 4 Key Steps:

(1) Extract/classify/filter/adjust/summarize data on bond prices from EJY:

- Classify by sector, risk-grade, and maturity
- remove outliers & redundant observations
- Strip out option values & adjust to a standard structure (e.g. 50% LIED)
- Summarize: zero term structure or averages by risk grade & maturity

(2) Fit the credit model to the summarized price data for each risk grade:

- Inputs: (1) indicative prices, coupons, & LIED rates by risk rating by term; (2) risk-free curve; (3) prior (empirical) transition matrix
- Output: term structure of RN transition matrixes (fit to benchmark prices)

(3) Determine generators that closely approximate the RN transition matrixes

(4) Validate the calibration using data outside the estimation sample

Calibration: Prior credit yield curves construction

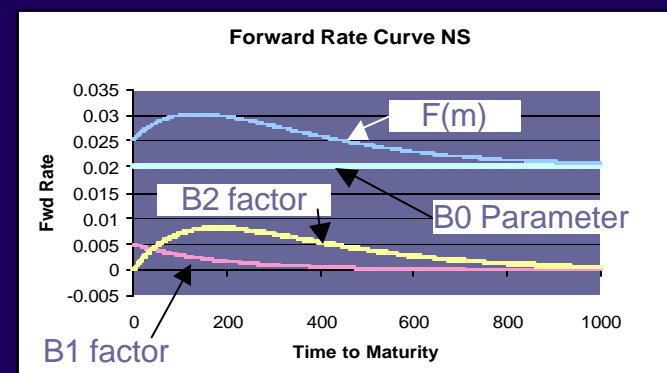
Prior credit yield curve construction:

- Input:
 - set of “basis” instruments per rating and sector and their prices
 - risk-free term structure
- Output:
 - “basic spread matrix”: term structure of Zero-prices per rating & sector
- Module:
 - calibration libraries with yield curve models: Intensity based models, Nelson-Siegel, Svenson, B-Splines, bootstrapping
- Main objective: stripping of bond coupons and robust statistical estimation of zeros at standardized terms (can also extend to longer terms than observed)

Example: Nelson Siegel Smoothing Curves

Defined by the equation for the Instantaneous Forward Rates :

$$f(m) = \beta_0 + \beta_1 \cdot \left(e^{-\frac{m}{t}} \right) + \beta_2 \cdot \frac{m}{t} \cdot \left(e^{-\frac{m}{t}} \right)$$



m , term to maturity.

β_0 , asymptotic value of the forward rate (as m goes to infinity)

β_1 , short-term value of the curve minus the asymptotic value

- $\beta_0 + \beta_1$ is the is the interception with the vertical axis.

β_2 , concavity or convexity of the curve and its magnitude .

- If positive, a concavity will occur at τ ; if negative, a convexity value will occur at τ .

t , mean-reverting parameter (indicates where the convexity or concavity will occur)

Zero Nelson Siegel Curve

Integrating we can obtain the Zero Nelson Siegel Curve:

$$Z(m) = \beta_0 + \beta_1 \cdot \frac{t}{m} \cdot \left(1 - e^{-\frac{m}{t}}\right) + \beta_2 \cdot \left(\frac{t}{m} \cdot \left(1 - e^{-\frac{m}{t}}\right) - e^{-\frac{m}{t}}\right)$$

This model is used to obtain the term structure of the Risk Free Interest Rate and the Zero + Spread curves for the different ratings ranges.

Some Issues:

- Handling coupon bonds and stripping coupons
- Must standardize prices by LIED
- Zero rates could cross from one rating curve to other rating curve

Fitting Process

$$Z(m) = \beta_0 + \beta_1 \cdot \frac{t}{m} \cdot \left(1 - e^{-\frac{m}{t}}\right) + \beta_2 \cdot \left(\frac{t}{m} \cdot \left(1 - e^{-\frac{m}{t}}\right) - e^{-\frac{m}{t}}\right)$$

General Problem: nonlinear function $F(m, \tau)$; fitting the market data to this model leads us a Nonlinear Optimization Problem with four parameters: 3 Betas and 1 Tao.

Fixing τ , the problem is simplified to a Least Squares Optimization with linear constraints.

One simple solutions process:

1. Find the best fit for the Risk Free Rate solving for the Four parameters including Tao, with a nonlinear optimization.
2. Use the same Tao for all the following credit curves and with a Least Squares Optimization find the Betas for this new curves.
3. Linear Constrains

- $R_i(m) \leq R_j(m), \quad \forall j < i; \text{ and } \quad \forall m;$

where j and i are the credit rating index

- β_0 , is greater than zero.
- $\beta_0 + \beta_1$ is also, greater than zero.

Nelson Siegel

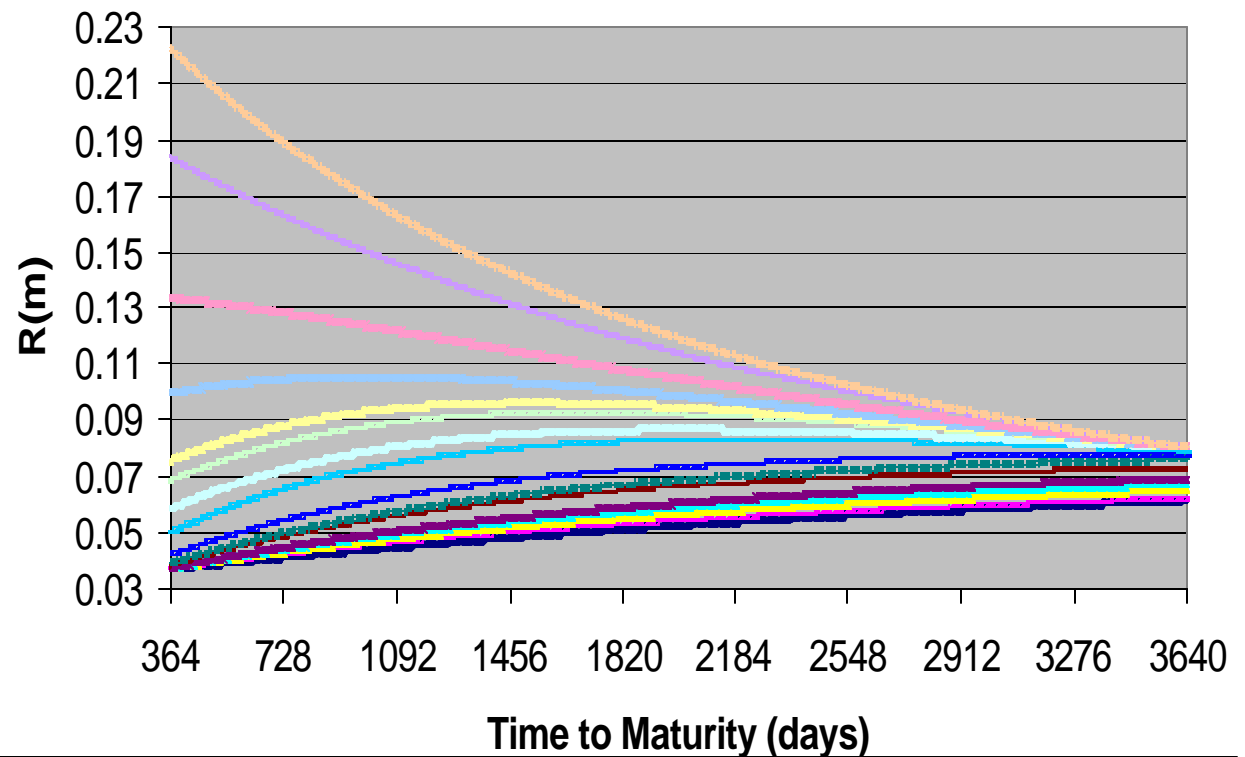
Example Results (fit

Lied = 50%

Tao = 1025

Beta_0	Beta_1	Beta_2
0.079188	-0.04649	-0.02114
0.07578	-0.04462	-0.00633
0.07993	-0.0498	-0.00488
0.080464	-0.05187	0.003749
0.084094	-0.05565	0.002415
0.077541	-0.05392	0.04093
0.082535	-0.05804	0.039503
0.074967	-0.05048	0.06932
0.049027	-0.02388	0.14942
0.045749	-0.0106	0.15327
0.032457	0.012041	0.18227
0.027271	0.026457	0.18735
0.026128	0.061553	0.15281
0.023022	0.11333	0.10766
0.019448	0.18794	0.038893
0.020886	0.2442	-0.02983

Credit Spreads Curves



Calibration: Multi-State Model

- Input:
 - “Basic spread matrix” (term structures of Zero-prices)
 - Real transition matrix (prior)
- Output
 - term structure of risk-neutral transition matrices
 - “smoothed spread matrix”
- Module: solution of “global” optimization problem with structure constraints
- Requirements:
 - flexibility in LGD model (RoT, RMV, RoP), TM transformation (JLT, KK, CM), weight setting, constraints
 - allow for coupon instrument calibration
 - robust estimation of generators (Transition manager)

The Multi-State Credit Model Calibration Problem

- Calibrate a multiple state credit model to existing market prices
 - given assumptions about payments in each state of the world → credit migration probabilities under the chosen martingale pricing measure
- Resulting migration probabilities must take sense
 - migration probabilities must be between zero and one
 - probabilities of default must increase with decreasing credit quality
- Main issues:
 - too many parameters → need to define lower dimensional model
 - difficult to enforce structure with “standard” bootstrap calibration

Goals of Calibration Framework

- Maximum flexibility in choice of
 - base calibration instruments (swaps, coupon bonds, etc.)
 - recovery assumptions (RP, RT and credit RMV)
 - migration transformations (low-dim. model) (Credit Metrics, JLT, KK...)
- Robust to handle possibly noisy input prices
 - allow to incorporate beliefs about structure of transition matrix (under the martingale pricing measure)
- Migration model must be internally consistent
 - calibrate all credit ratings together and all terms simultaneously rather than in an independent fashion (as in a bootstrap calibration approach)

The Framework

- Define a norm $\| \hat{\Pi}, \Pi(Q_T) \|$
 - the *distance* between the *observed* market prices $\hat{\Pi}$ and the prices $\Pi(Q_T)$ corresponding to an element $Q_T \in \mathbf{Q}_T$
- Calibration problem can be formulated as

$$\min_{Q_T \in \mathbf{Q}_T} \| \hat{\Pi}, \Pi(Q_T) \|^2$$

(Note that we'll do at least as well as a bootstrap approach)

- Important practical issue: assumption of independence of the credit migration process and the riskless rates (under martingale measure)
 - necessary for computational tractability of multi-step model

Dimensionality Reduction

- Problem: calibration instruments do not provide sufficient information
 - In practice: not rich enough payoffs in different credit states to uniquely determine the optimal solution

$$Q_T^* \in Q_T$$

- Solution: specify a subset $\tilde{Q}_T \subseteq Q_T$ and solve

$$\min_{Q_T \in \tilde{Q}_T} \|\hat{\Pi}, \Pi(Q_T)\|$$

- Choose $\tilde{Q}_T \subseteq Q_T$ so as to
 - Reflect our beliefs about the *structure* of migration probabilities
 - Achieve a desirable tradeoff between *speed* and *accuracy*

Migration Transformations

- Often the structure of our base calibration instruments only provides sufficient information to determine the default probabilities

$$\wedge \in \mathfrak{R}_{[0,1]}^{k \times N}$$

- Assume the existence of a transformation $G(\cdot; P, \hat{\Pi})$ such that

$$G(\wedge) = \mathcal{Q}_T = \{\mathcal{Q}(t_{n-1}, t_n)\}_{n=1}^N \in \mathbf{Q}_T$$

- Choice of transformation can *indirectly* reflect *utility* preferences; e.g.
 - Jarrow, Lando, Turnbull (1997)
 - Kijima and Komoribayashi (1998)
 - One-factor structural model (Aguais et al)

Important practical issues

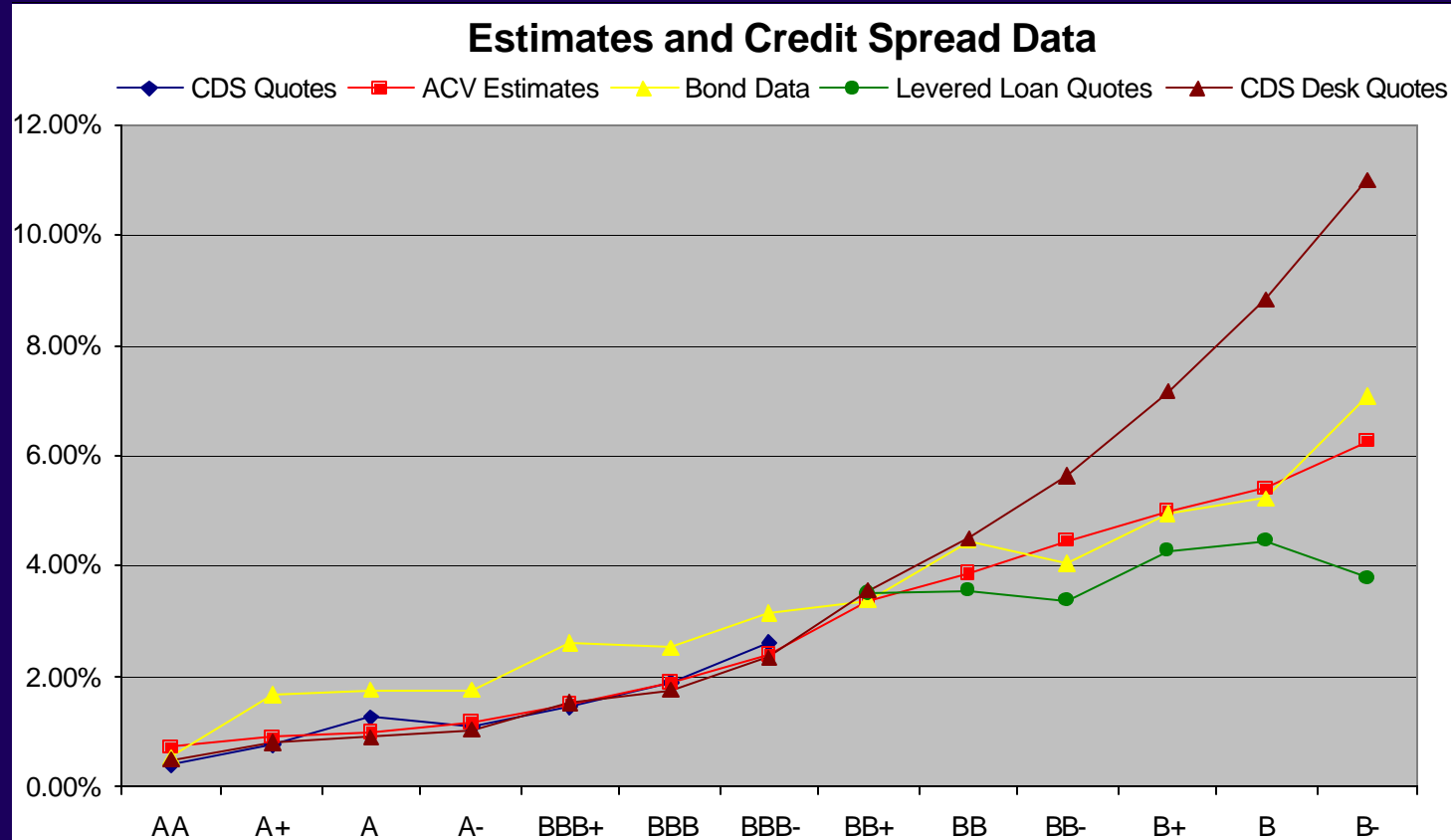
- Optimization problem may be difficult to solve:
 - powerful tools required (e.g. successive relaxed parameterization)
- Extremely difficult to get reliable input prices
 - Instruments with embedded options (e.g. callability or putability) are difficult to use as base instruments in calibration
- Need to compute *Generators* (e.g. to price intermediate payoffs between the maturities we have calibrated to)
 - It may be more efficient to parameterize directly the generators
- Note that, in addition to the credit independence assumption, credit spreads for each rating at each future point in time are assumed deterministic

- Simple extension to stochastic forward credit spreads
 - achieved by allowing (real) migration probabilities to be stochastic and depend on an additional independent factor (other than credit rating; e.g. a systemic credit risk factor from a structural model)
 - The choice of deterministic versus stochastic forward credit spreads should reflect the definition of credit states
- Choose our migration transformation to match a set of moment conditions (possibly include second moments and more, not just first moments)
 - calibrate to volatilities (and other known conditions of the data)

Example: Baseline 5-Year Curve & Related Data for January 2002

In practice must make fitting decision explicit; e.g.

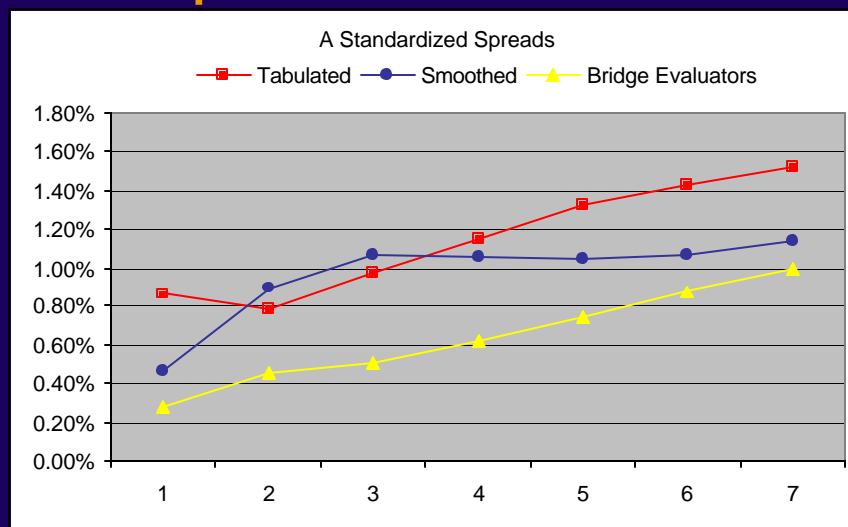
- baseline fit to investment-grade CDS quotes & high-yield bond prices
- compromise among various credit “price” information sources



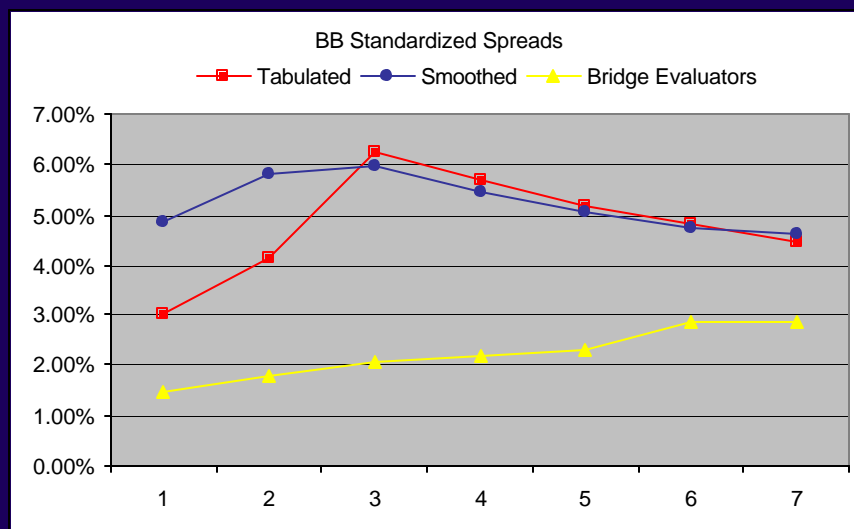
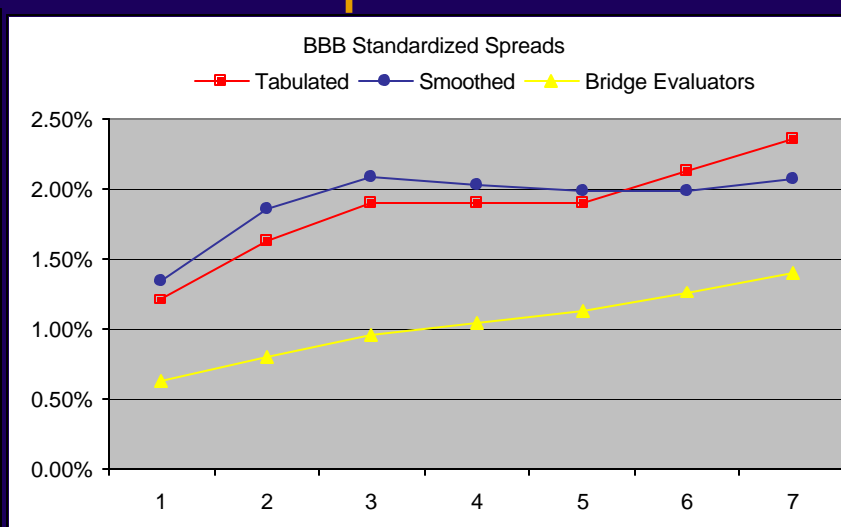
3 Views of Credit Spreads: Smoothed, Raw Tabulated & Bridge Evaluator's

Algorithmics
Incorporated 

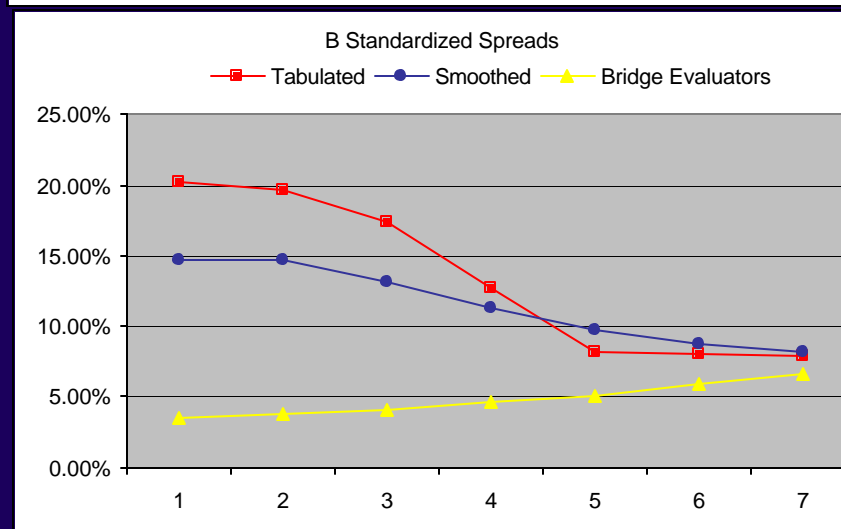
A Spreads:



BB Spreads:



BBB Spreads:



B Spreads:

Empirical Transition Matrix

Real, Empirical One-Year Transition Matrix (Developed from KMV Data)

Real Empirical Transition Matrix																	
	1	1-	2+	2	2-	3+	3	3-	4+	4	4-	5+	5	5-	6	7	D
AAA	71.81%	17.67%	4.71%	2.53%	0.93%	0.99%	0.59%	0.33%	0.20%	0.10%	0.06%	0.04%	0.02%	0.01%	0.01%	0.00%	0.00%
AA	24.28%	48.71%	13.00%	5.44%	4.13%	1.80%	1.09%	0.62%	0.39%	0.21%	0.13%	0.09%	0.05%	0.03%	0.01%	0.00%	0.02%
AA-	0.03%	29.84%	42.14%	9.80%	9.44%	3.60%	2.08%	1.47%	0.66%	0.37%	0.14%	0.19%	0.11%	0.05%	0.03%	0.02%	0.03%
A+	0.02%	9.70%	21.12%	42.02%	12.85%	6.38%	4.65%	1.57%	0.74%	0.37%	0.14%	0.19%	0.11%	0.04%	0.03%	0.02%	0.03%
A	0.02%	1.40%	13.97%	19.92%	43.42%	9.54%	6.91%	2.31%	1.10%	0.55%	0.21%	0.28%	0.15%	0.06%	0.04%	0.03%	0.08%
A-	0.05%	1.71%	4.61%	10.12%	14.33%	40.45%	11.88%	8.95%	6.20%	0.68%	0.28%	0.40%	0.13%	0.05%	0.03%	0.02%	0.12%
BBB+	0.03%	1.02%	2.10%	5.59%	8.64%	16.64%	39.86%	13.04%	9.30%	2.27%	0.44%	0.60%	0.18%	0.07%	0.03%	0.02%	0.14%
BBB	0.02%	0.86%	1.81%	4.86%	7.74%	6.12%	10.56%	42.08%	13.92%	6.21%	3.52%	1.26%	0.46%	0.21%	0.04%	0.02%	0.30%
BBB-	0.03%	0.40%	0.47%	0.85%	1.43%	2.64%	4.91%	9.75%	42.92%	13.10%	11.19%	9.19%	1.40%	0.91%	0.21%	0.01%	0.59%
BB+	0.02%	0.22%	0.26%	0.47%	0.80%	1.46%	2.76%	5.61%	9.42%	42.59%	15.03%	13.37%	5.21%	1.49%	0.37%	0.01%	0.90%
BB	0.01%	0.11%	0.13%	0.23%	0.39%	0.72%	1.32%	2.66%	4.55%	8.40%	42.41%	17.28%	11.30%	8.14%	0.70%	0.01%	1.66%
BB-	0.00%	0.05%	0.06%	0.11%	0.18%	0.34%	0.62%	1.16%	2.10%	3.91%	7.34%	55.64%	12.67%	9.91%	3.38%	0.01%	2.51%
B+	0.00%	0.05%	0.05%	0.10%	0.16%	0.30%	0.55%	1.03%	1.84%	3.41%	6.37%	10.92%	54.88%	11.86%	3.98%	0.01%	4.48%
B	0.00%	0.04%	0.05%	0.09%	0.15%	0.28%	0.50%	0.93%	1.66%	3.06%	5.70%	9.70%	9.40%	53.68%	7.07%	0.02%	7.65%
B-	0.00%	0.03%	0.04%	0.07%	0.11%	0.21%	0.38%	0.70%	1.26%	2.34%	4.13%	7.42%	7.31%	7.02%	56.13%	0.11%	12.75%
CCC+	0.00%	0.03%	0.04%	0.07%	0.11%	0.20%	0.37%	0.67%	1.21%	2.24%	3.97%	7.05%	6.91%	6.60%	5.17%	49.22%	16.15%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

Risk-Neutral Transition Matrix

Forward Transition Matrix for Year 3 - Risk Neutral Needed for Valuation

- Calibration Adjusts default rates to reflect both Risk Premiums & Expected Losses
- One year BBB+ Default Rate - 14bp (from empirical transition matrix)
- Year Three One-Year Forward Risk Neutral Default Rate - 82bp

Year 3	AAA	AA	A+	A	A-	BBB+	BBB	BBB-	BB+	BB	BB-	B+	B	B-	CCC+	CCC	D
AAA	50.42%	24.94%	8.87%	5.63%	2.32%	2.75%	1.84%	1.17%	0.77%	0.45%	0.29%	0.22%	0.13%	0.08%	0.04%	0.02%	0.04%
AA	9.12%	39.94%	18.08%	9.63%	8.89%	4.66%	3.27%	2.12%	1.49%	0.92%	0.64%	0.48%	0.29%	0.20%	0.10%	0.00%	0.16%
A+	0.00%	12.21%	35.67%	12.84%	15.71%	7.57%	5.15%	4.28%	2.26%	1.41%	0.60%	0.88%	0.56%	0.27%	0.21%	0.14%	0.26%
A	0.00%	2.66%	10.13%	36.11%	17.79%	11.40%	10.55%	4.51%	2.51%	1.44%	0.59%	0.88%	0.55%	0.26%	0.20%	0.14%	0.29%
A-	0.00%	0.23%	4.65%	10.67%	40.85%	14.57%	13.79%	5.94%	3.35%	1.93%	0.81%	1.17%	0.73%	0.32%	0.24%	0.17%	0.57%
BBB+	0.00%	0.30%	1.23%	3.82%	7.41%	34.24%	15.69%	15.40%	15.02%	2.28%	1.03%	1.65%	0.61%	0.26%	0.14%	0.10%	0.82%
BBB	0.00%	0.16%	0.47%	1.68%	3.45%	8.98%	35.42%	18.53%	18.64%	6.42%	1.51%	2.33%	0.83%	0.34%	0.17%	0.12%	0.96%
BBB-	0.00%	0.13%	0.39%	1.40%	2.93%	2.82%	5.81%	36.88%	20.10%	12.04%	8.83%	4.01%	1.76%	0.90%	0.17%	0.11%	1.72%
BB+	0.00%	0.08%	0.11%	0.23%	0.43%	0.93%	2.03%	4.93%	33.73%	15.07%	15.94%	17.36%	3.48%	2.60%	0.68%	0.04%	2.37%
BB	0.00%	0.03%	0.04%	0.09%	0.17%	0.36%	0.81%	2.01%	4.25%	31.68%	17.62%	21.45%	11.79%	4.37%	1.26%	0.05%	4.02%
BB-	0.00%	0.02%	0.03%	0.06%	0.11%	0.22%	0.46%	1.05%	2.09%	4.55%	33.75%	19.78%	16.21%	15.18%	1.63%	0.03%	4.85%
B+	0.00%	0.01%	0.02%	0.03%	0.06%	0.13%	0.25%	0.51%	1.02%	2.11%	4.49%	49.47%	15.73%	14.64%	5.96%	0.02%	5.53%
B	0.00%	0.01%	0.01%	0.01%	0.03%	0.06%	0.12%	0.26%	0.53%	1.16%	2.63%	5.61%	49.57%	18.75%	8.12%	0.03%	13.11%
B-	0.00%	0.00%	0.01%	0.01%	0.03%	0.05%	0.11%	0.23%	0.48%	1.04%	2.32%	4.84%	5.65%	53.71%	12.05%	0.04%	19.40%
CCC+	0.00%	0.01%	0.01%	0.02%	0.03%	0.07%	0.13%	0.27%	0.53%	1.09%	2.17%	4.43%	4.91%	5.16%	58.34%	0.16%	22.68%
CCC	0.00%	0.01%	0.01%	0.02%	0.03%	0.07%	0.13%	0.26%	0.51%	1.05%	2.08%	4.19%	4.60%	4.79%	4.01%	50.92%	27.32%
D	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	100.00%

More Detailed Comparison With Syndicated Loan Prices

January bond calibration understates selected, leveraged loan prices by an average of 135 bps

Name	Facility	Rating	Maturity	GS/LPC	ACV	Diff
Adelphia	TLb	BB-	6/30/09	95.75	94.91	0.84
Argosy Gaming	TLb	BB	3/31/06	100.63	98.43	2.19
Armkel	TLb	B+	3/28/09	100.25	97.93	2.32
Ball Corp	TLb	BB	3/10/06	100.25	98.34	1.91
Broadwing	TLb	B	12/30/06	95.00	95.99	-0.99
Charter Corp	TLb	BB	3/18/08	99.00	95.85	3.15
DRS Technologies	TLb	BB-	9/30/08	100.75	98.27	2.48
Extended Stay Americas	TLb	BB-	1/15/08	100.00	98.00	2.00
Flowers Foods	TLb	BBB-	5/1/07	100.25	100.29	-0.04
Insight Midwest	TLb	BB	12/31/09	100.50	99.17	1.33
Isle of Capri	TLb	BB-	2/2/06	100.50	99.19	1.31
Levi Strauss	TLa	BB-	8/29/03	99.50	100.81	-1.31
Magellan Health	TLb	B+	2/15/05	99.63	96.68	2.94
SPX	TLb	BB+	9/30/06	100.00	98.09	1.91
Stone Container	TLf	B+	12/31/05	100.00	98.24	1.76
Suiza Food	TLb	BB	7/15/08	101.00	100.19	0.81
Volume Services	TLb	B+	12/31/06	98.50	98.10	0.40
Werner Holding	TLb	B+	11/30/04	98.00	97.66	0.34
Werner Holding	TLc	B+	11/30/05	98.00	96.24	1.76
Willis Corroon	TLc	BB	2/19/08	99.50	97.59	1.91
Average			2/17/07	99.35	98.00	1.35
Median		BB-	12/30/06	100.00	98.10	1.91
Average tenor			5 years			
Correlation: ACV vs. Market				0.71		

Transition Matrices

- Pricing in practice requires the computation of transition probabilities over time intervals of less than one year.
- In a majority of practical cases, the annual transition matrix A does not have a generator (root matrices might not be real).
- Solution: solve regularization problem - Find a transition matrix X that, when raised to the power t , most closely matches the annual matrix A .
 - *Problem BAM (Best approximation of the annual transition matrix)*

Find $\tilde{X} \in TM(n)$ such that

$$\|\tilde{X}^t - A\| = \min_{X \in TM(n)} \|X^t - A\|$$

where $\|\cdot\|$ is a suitable norm in the space of $n \times n$ matrices.

Problem BAM is a high-dimensional, constrained non-linear optimization problem whose solution is computationally intensive.

Transition Matrices

- Some Practical heuristics necessary to solve this difficult problem in practice

Problem QOM: Quasi-optimization of the root matrix

Find $\hat{X} \in TM(n)$ such that

$$\|\hat{X} - A^{1/t}\| = \min_{X \in TM(n)} \|X - A^{1/t}\|$$

Problem QOG: Quasi-optimization of the generator

Find $\hat{G} \in G(n)$ such that

$$\|\hat{G} - \ln(A)\| = \min_{X \in G(n)} \|X - \ln(A)\|$$

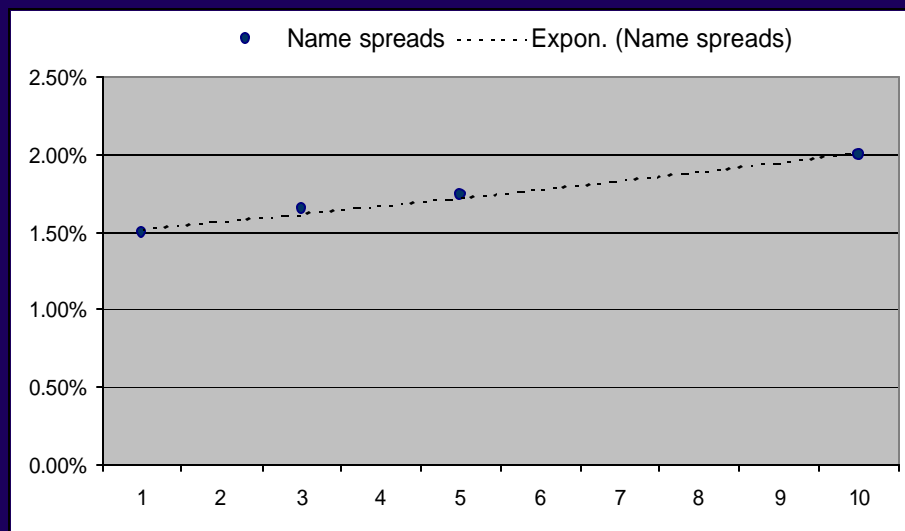
- Problems QOM or QOG are much more computationally attractive than problem BAM and their solutions should be close;
- refer to these solutions as quasi-solutions to problem BAM

- Input
 - “Smoothed spread matrix” (term structures of Zero-prices)
 - probably with some measures on dispersion of specific spreads
 - RN transition matrix
 - Name specific Zero terms structure
- Output
 - term structure of Name risk-neutral transition matrices
 - “smoothed Name spread matrix”
- Module: mathematical formulation of specific risk term structure assumptions; solution of “global” optimization problem with structure constraints;
- Requirements: flexibility in “specific risk model”, LGD model, transformation, weight setting, constraints; allow for coupon instrument calibration

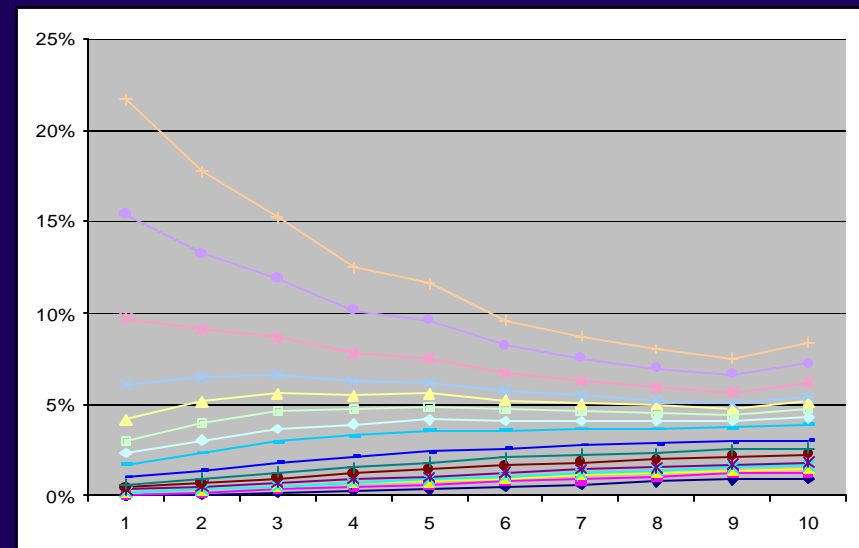
Name Calibration: Solution with weighted Baseline Curves

Find the weighted average of baseline credit curves that provide the best fit to the single-name curve:

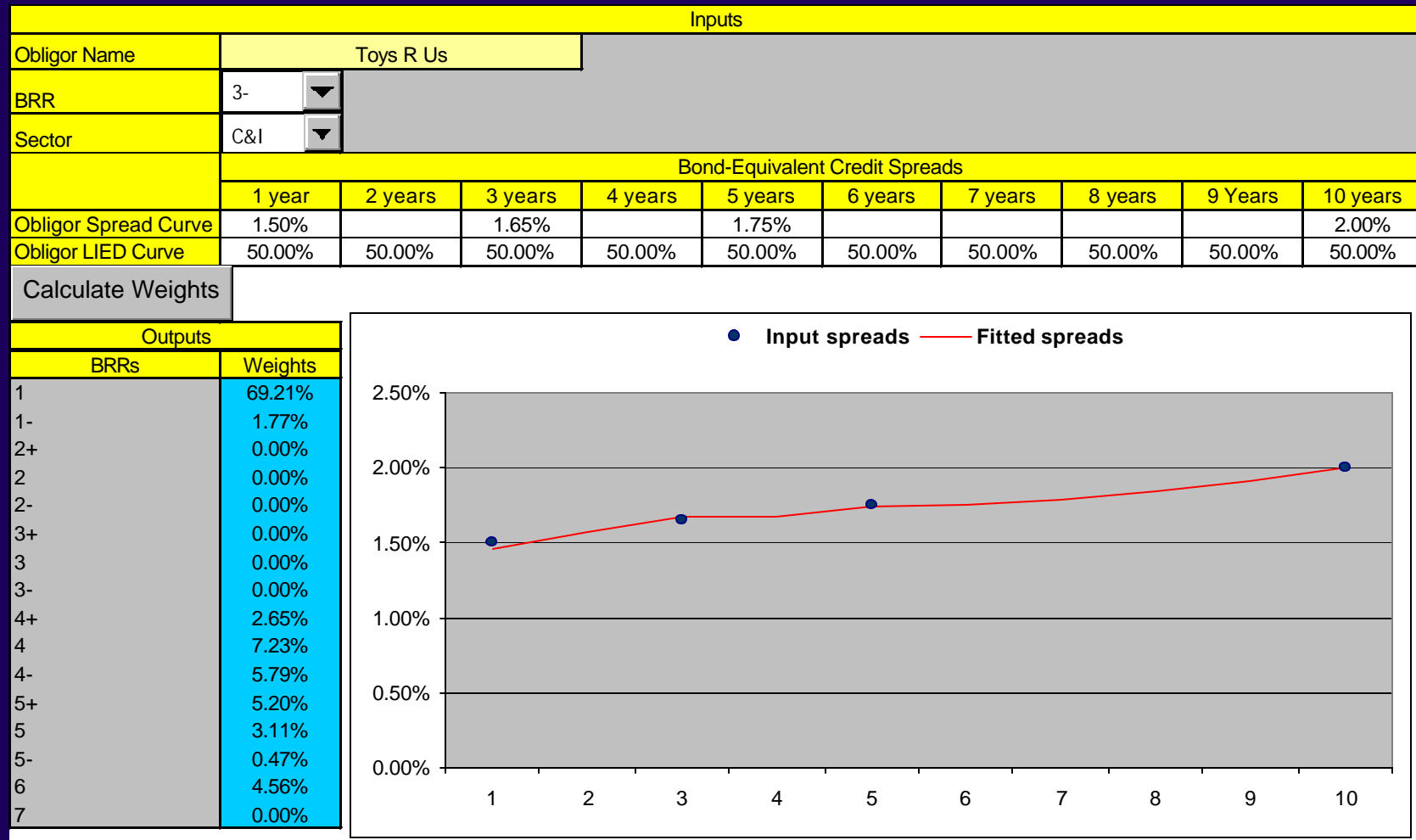
Name curves often only a few points.



Baseline curves for different levels of credit worthiness provide a wide range of shapes & levels.



Name Calibration: Solution with weighted Baseline Curves



Outline

- Enterprise credit risk management & valuation
- Loan Valuation and MtM
 - MtM of Loans
 - properties & embedded options
 - underlying credit model
- Loan valuation Framework
- Calibration in practice
- Examples

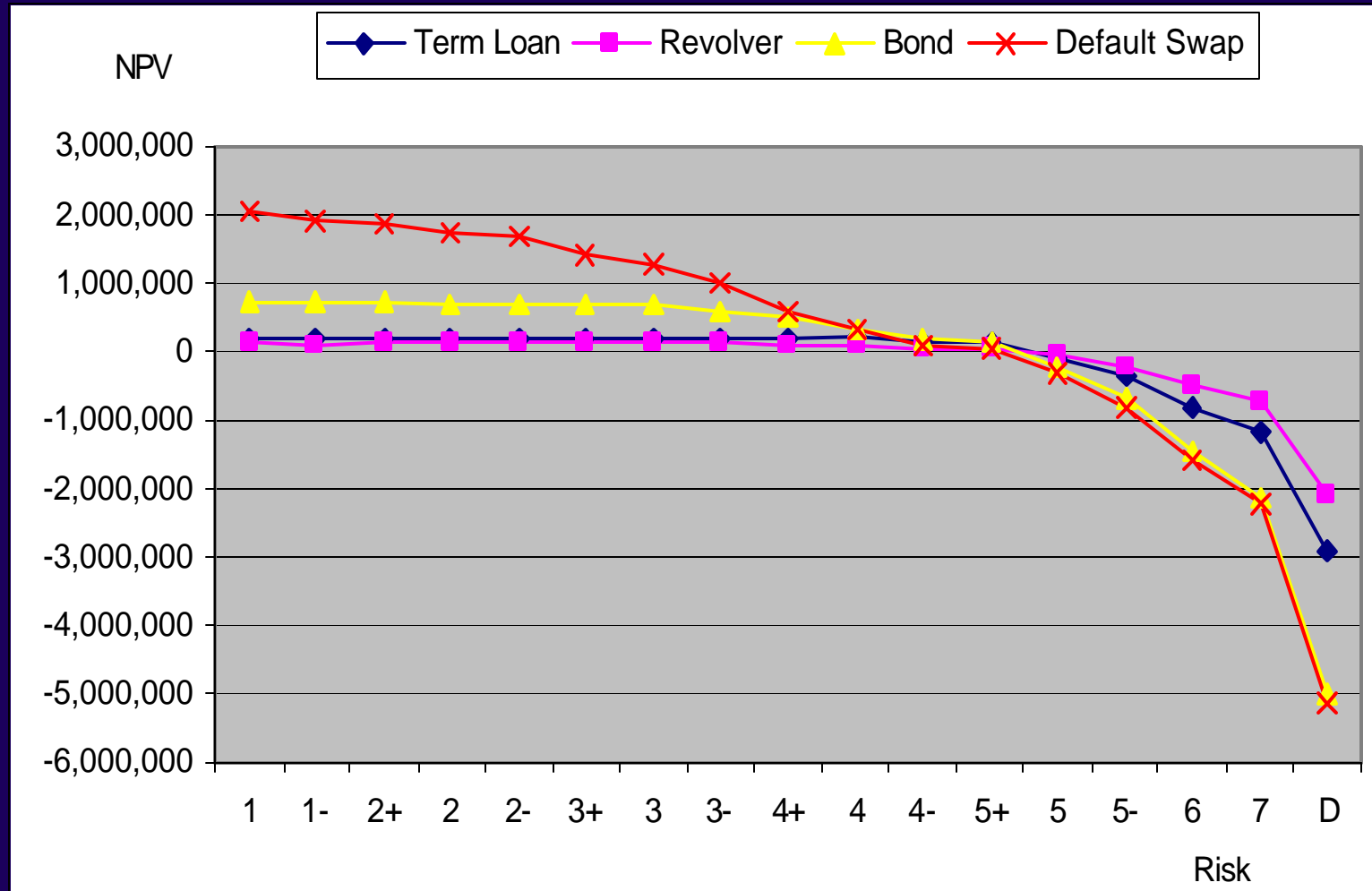
Example 1: Evaluating Hedge Effectiveness

- Loans, Bonds & Credit Derivatives Exhibit Highly Non-Linear Responses to Changes in Creditworthiness
- Consider \$10 million Notional Positions in the Following Four Distinct Credit Facilities With the Same B+ (BRR = 5+) Obligor:
 - **Term Loan:** Maturing on 11/30/04, with grid pricing, a variable amortization schedule & accounts receivable collateral
 - **Revolver With LC Option:** Maturing on 11/24/03, with grid pricing and accounts receivable collateral
 - **Senior, Unsecured Bond:** Maturing on 11/17/07, with a 10 percent annual coupon, payable semi-annually, callable for the first time on 11/17/02 and callable thereafter every 6 months
 - **Credit Default Swap:** With a semi-annual swap payment of 445 bps annually, with the above bond or an available substitute in the case of prepayment of the underlying reference asset

Evaluating Hedge Effectiveness:.. .

- Four Example Credit Facilities Vary By:
 - Time-to-maturity
 - Payment dates
 - Lined rates
 - Embedded optionality (prepayment, line utilization, grid pricing, covenants etc.)
 - Liquidity influences
 - Interest rate risk
- Hedging the term loan with any of the other three positions is quite complicated

Valuing Credit Instruments after One Quarter Under Grade Migration



Example 2: Pre-Deal Pricing & Structuring

Argosy Gaming -- BB Rating
\$10 M Term Loan, 7-Year Maturity
Senior Secured, Grid priced,
Back-loaded amortization,
Callable without penalty

Base Case:

NPV (\$)	-88,525
Price (% par)	99.11
Duration (years)	3.44

250 bps Call Premium for 4 years:

NPV (\$)	-60,260
Price (% par)	99.4
Duration (years)	4.19

Plus faster amortization (SL after yr 4)

NPV (\$)	-9,875
Price (% par)	99.90
Duration (years)	4.24

Agco Corp. -- BB Rating
\$35 M Revolver/LC
4 1/2-Year Maturity
Senior Secured, Grid priced,
Bullet, Callable without penalty

Base Case:

NPV (\$)	-43,817
Price (% par)	99.17
Duration (years)	3.44

Add 25 bps step-up at $\leq B^+$ & cut
spreads on step-downs by 25 bps

NPV (\$)	33,049
Price (% par)	100.63
Duration (years)	3.12

Example 3: Valuation Case Study

- Used model to assess a portfolio of 121 credit facilities:
 - Investment grade & leveraged loans
 - Data supplied by PMD & LPC Gold Sheets
 - Case Study Portfolio: 6 different facility/product types
 - Assumed “hold” levels for each public tranche
 - Portfolio covered 7 different industry sectors
 - 6 Downgrades & 4 Upgrades
 - Used Bridge/EJV to develop two ACV C&I calibrations:
 - September 1 & November 1, 2001
 - Ran ACV in MtM (batch-mode) Sept1 & Nov 1, 2001
 - **Allows a MtM assessment pre & post-September 11th**

Example 3: Valuation Case Study

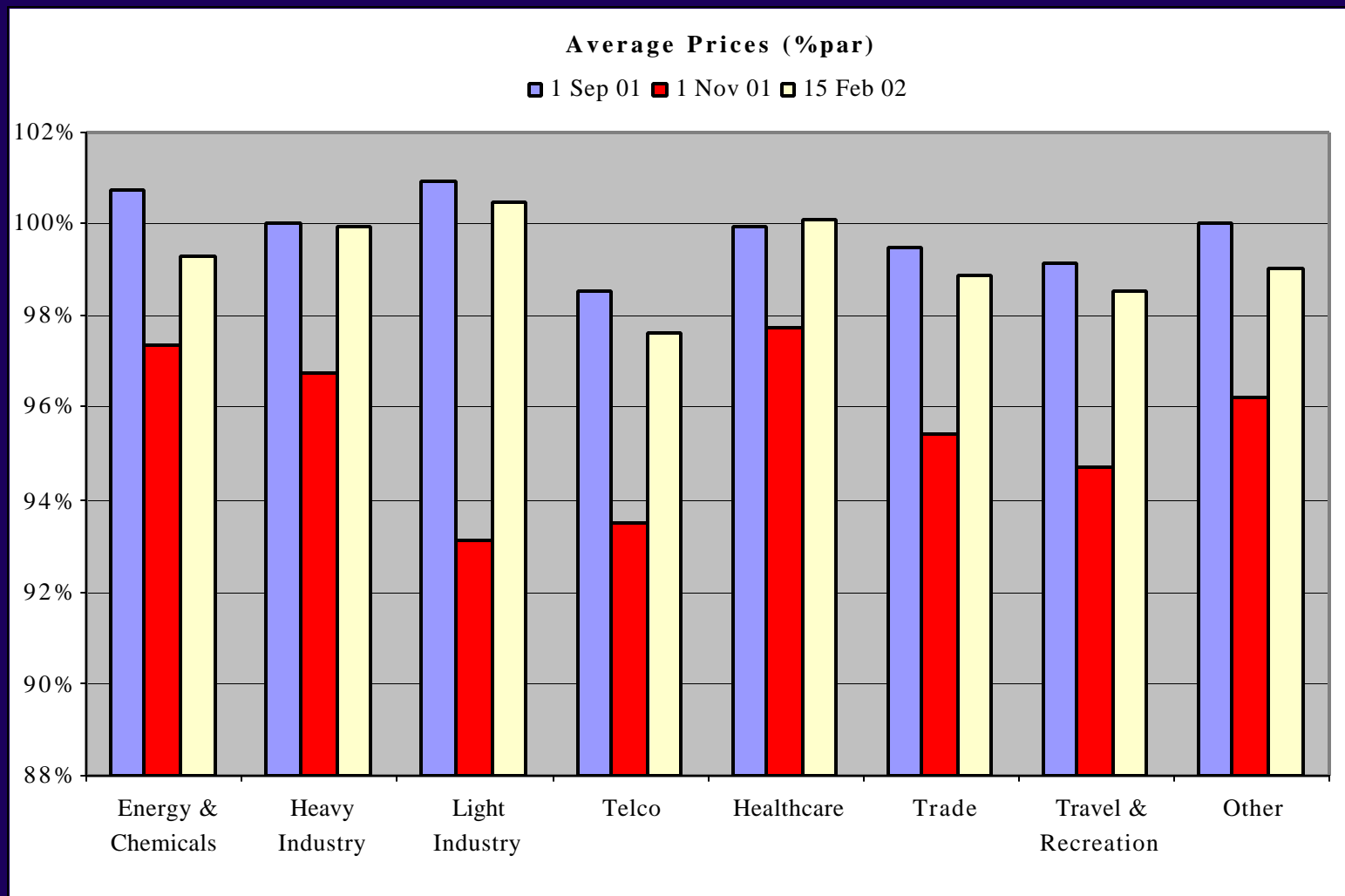
- Portfolio of 121 credit facilities:
 - Investment grade & leveraged loans
 - Data supplied by PMD & LPC Gold Sheets
 - Case Study Portfolio: 6 different facility/product types
 - Assumed “hold” levels for each public tranche
 - Portfolio covered 7 different industry sectors
 - 6 Downgrades & 4 Upgrades
 - Used Bridge/EJV to develop three ACV calibrations:
 - September & November 2001 and February 2002
 - MtM in Sept 1 & Nov 1, 2001 & Feb 15, 2002
 - Compare MtM assessment pre & post-September 11th

Portfolio Composition

<u>Product</u>	<u># Facilities</u>	<u>Total Commitments</u>
• Term loans:	45	\$463.9 million
• Revolvers :	37	\$1,002.1 million
• Revolver/LCs:	31	\$553.2 million
• Default Swaps:	1	\$10.0 million
• Bonds:	6	\$60.0 million
• LC:	1	\$15.0 million
 TOTAL PORTFOLIO: billion	 121	 \$2.104

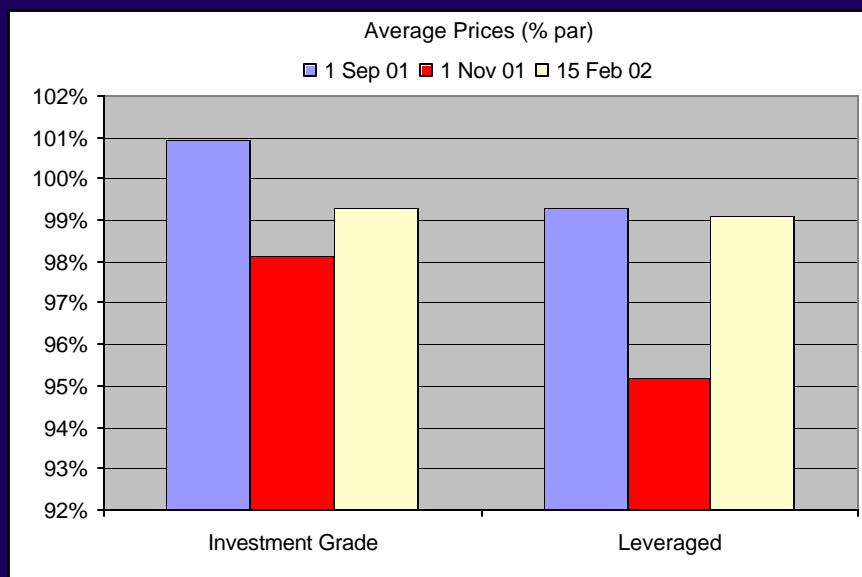
MtM Valuation - Prices Fell In Sep and Nov and Have Partly Recovered by Feb 15

Price (% PAR) by Industry Sector

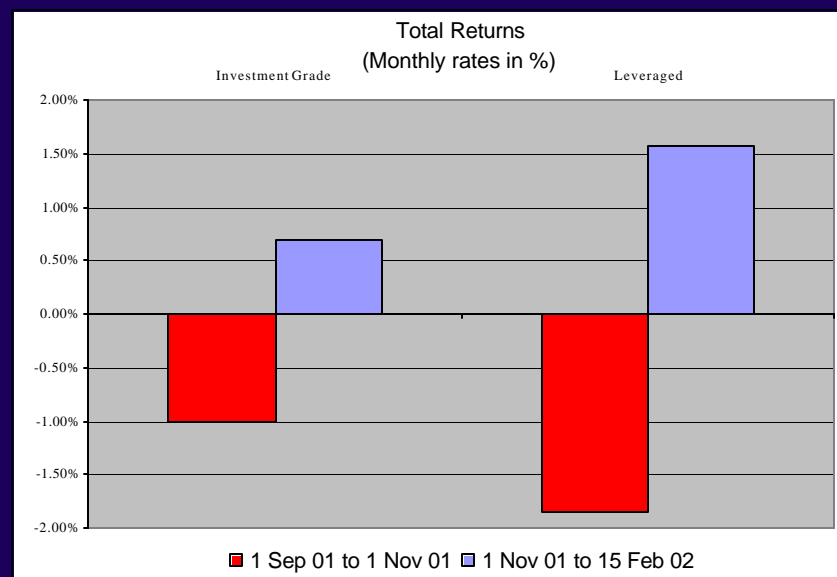


Investment Grade vs. Leveraged Portfolios— More Volatility in Leveraged

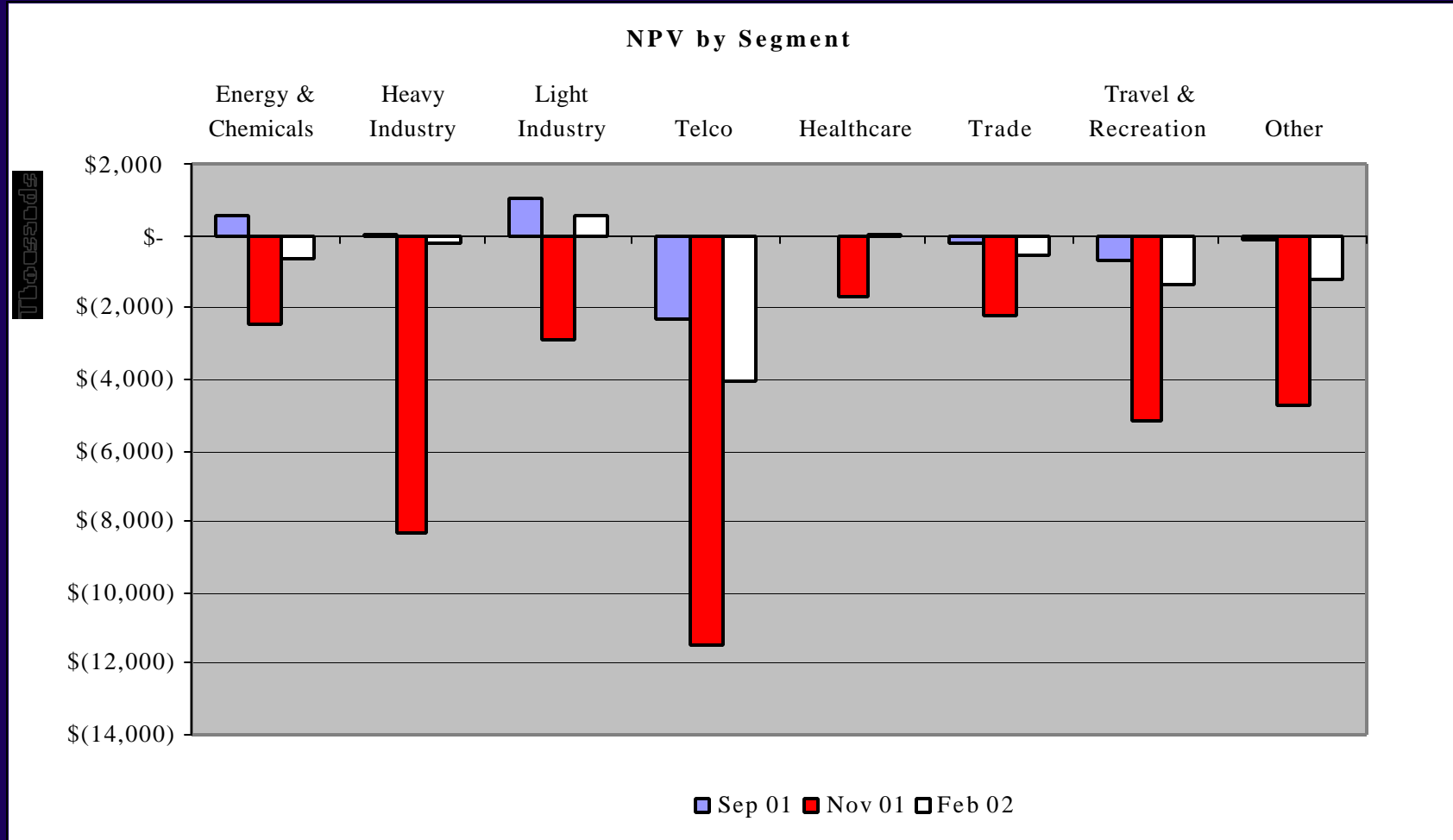
Average Prices Fell then Recovered



Total Returns Were Negative

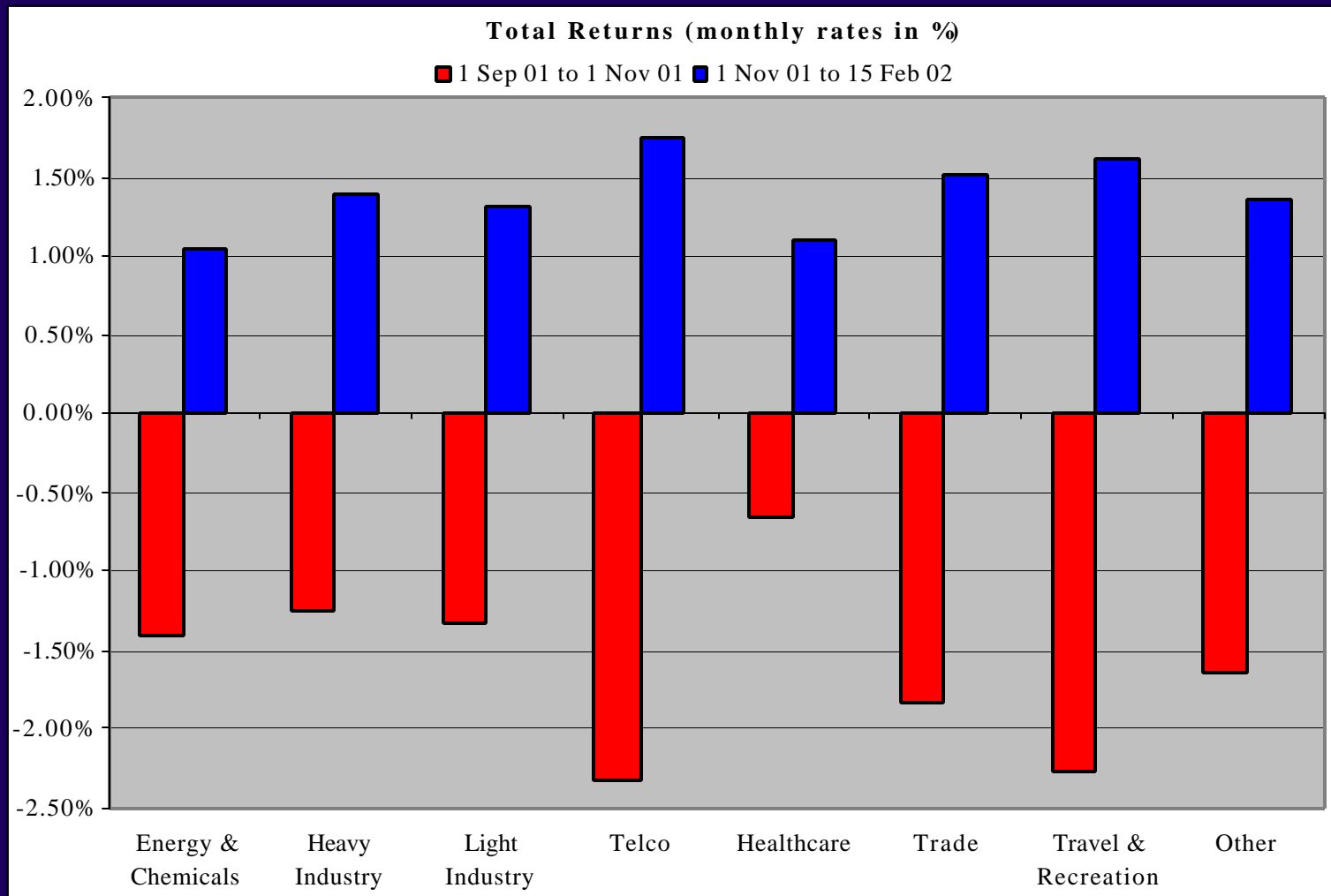


After Sept 11th -- MtM Valuations (NPV) Fell Across the Board



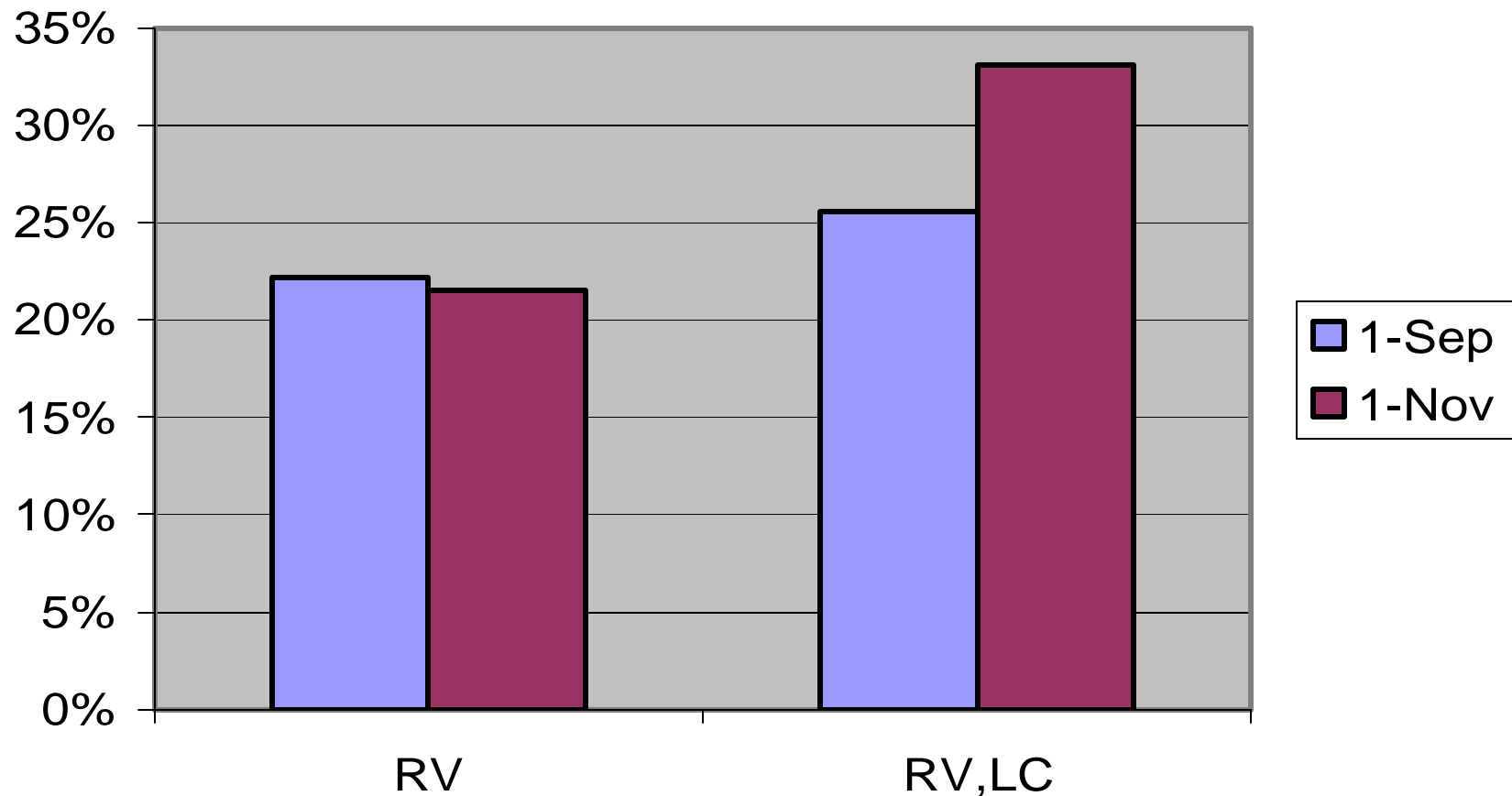
After Sept 11th -- Total Returns (Sector) from Sept-Nov Were Negative

Total Returns Fell Across the Board and More Recently Have Turned UP



Predicted Utilization on Revolvers & Revolver/LCs From Sept to Nov 2001

Line Utilization by Product (% Authorized Commitment)



Stress Testing MtM Valuations & Assessing Prepayment Optionality

- Risk Analysis: Used the Nov 1, 2001 Valuation
 - Move all graded up & down by one notch & Re-Value

NPV

- Notch Up: \$(6,443,541)
- Current Rating: \$(18,065,817)
- Notch Down: \$(34,693,883)

- Prepayment Optionality:

	<u>As Contracted</u>	<u>No Prepayment</u>
• Sep 1, 2001:	\$(1,623,244)	\$6,410,351
• Nov 1, 2001:	\$(18,065,817)	\$(14,803,830)

Concluding Remarks

- Credit valuation plays a key role in enterprise credit risk management
 - pricing and structuring
 - dynamic management of portfolios
 - exploitation of arbitrage
 - portfolio credit risk modeling
- Credit valuation Framework
 - accurate modelling of structure
 - underlying credit model
 - calibration methodology - data
- Requires development of powerful computational tools to make it practical



www.algorithmics.com

www.mark-to-future.com

See also

- Enterprise Credit Risk with Mark-to-Future
- Algo Research Quarterly