

NEWSletter

DECEMBER 2001

DIRECTOR'S MESSAGE

The fall term is underway and the Institute is bustling with participants in the special year on Numerical and Computational Challenges in Science and Engineering, which this fall is focused on numerical analysis. Several grad courses and seminars associated with the program are occurring on a regular basis. In addition, there has been an interesting short course by Olavi Nevanlinna on the theory of meromorphic matrix valued functions, which combines matrix theory with the classical function theory developed and named after Olavi's great uncle, Rolf.

Another highlight of this program to date was the Coxeter Lecture Series in which Gene Golub spoke on "Matrices, Moments and Quadrature". This interesting series concentrated on numerical questions of computing certain functions of Hermitian matrices. The term

quadrature is an old term for evaluating integrals discretely. Such methods go back at least to Gauss, who observed that integrals of polynomials over a given interval can be accomplished by taking an appropriate linear combination of certain evaluations of the polynomial. Clever modifications of this technique allow good upper and lower estimates of other integrals. These techniques transfer well to the matrix setting, and were the starting point for Golub's talks.

An additional highlight of the fall semester was the presentation of the CRM-Fields Institute prize to Prof. William Tutte. Perhaps this was anticlimactic for him, as he had just received the Order of Canada in Ottawa the previous week. However he did not let on, and gave a talk where he explored some of his thinking, which led to the development of the Tutte polynomial for graphs. He side-stepped a question on

Director's Message continued on page 3

NUMERICAL AND COMPUTATIONAL
CHALLENGES IN SCIENCE AND ENGINEERING

The Thematic Year on Numerical and Computational Challenges in Science and Engineering (August 2001 - August 2002) is off to a great start. The first major event was the SciCADE 2001 International Conference on Scientific Computation and Differential Equations, co-sponsored by the Fields Institute, PIMS, NSERC, and SIAM and held in Vancouver from July 29 to

August 3. The focus of the meeting was the numerical solution of differential equations. The conference attracted over 200 participants from around the world.

Already this fall, we have held two small informal working groups at the Fields Institute, with about 20 participants each. The first, on Dynamics of Numerics, August 7 to 10, was orga-

Numerical Challenges continued on page 3

In this issue...

Director's message

Numerical and Computational Challenges in Science and Engineering

Conference and Memorial Service in Honour of George F.D. Duff

Coxeter Lecture Series: Gene Golub

First Annual McMaster Optimization Conference

Second Statistics Workshop at the University of Manitoba

Workshop on Numerical Linear Algebra in Scientific and Engineering Applications

International Conference on Scientific Computation and Differential Equations (SciCADE)

The Fields Institute Mathematics Education Forum

Call for Proposals, Nominations and Applications

Activities at the Fields Institute

The Early History of the Fields Medal

The Individual Finance and Insurance Decisions (IFID) Conference

Graduate Information Day

News from AARMS (Atlantic Association for Research in the Mathematical Sciences)

2001 CRM-Fields Prize Lecturer: William T. Tutte

A Brief History of Operator Algebras

Conference and Memorial Service in honour of Professor Emeritus George F.D. Duff, F.R.S.C.

On Friday, September 21, 2001, the Canadian Mathematical Society, the Department of Mathematics at the University of Toronto, the Fields Institute and the family of George Duff hosted a conference and memorial service in his honour at the Fields Institute.

George F.D. Duff was born in Toronto, July 28, 1926. He received his high school education at the University of Toronto Schools and undergraduate and Master's degrees (the latter under Leopold Infeld) from the University of Toronto. After obtaining a doctorate at Princeton under Solomon Lefschetz and spending a year at MIT as a Moore instructor, he returned to the University of Toronto in 1952 for the rest of this career. He retired in 1992, and passed away on March 2, 2001, at the age of 74.

George was an active research mathematician in the area of partial differential equations and their applications. His interests ranged from elliptic partial differential equations to wave phenomena and hyperbolic partial differential equations. He had a special interest in the use of the Bay of Fundy tides for power generation and received an LL.D. degree from Dalhousie in 1994 partially as a result of this work.

In his long and active career, he served the mathematical community in many different roles – among them, chair of his department, coorganizer of the International Congress of Mathematicians in 1974 in Vancouver (where he was also an invited speaker), editor of the *Canadian Journal of Mathematics*, and president of the CMS and honorary president of the Ontario Association of Mathematics Teachers (which reflected his role in reforming the Ontario mathematics curriculum and coauthoring a series of high school texts).

A more detailed biography, as well as reminiscences by colleagues and former students, can be found in the May, 2001 issue of the CMS Notes, which is available on the website journals.cms.math.ca/Notes/.

The Conference

The conference consisted of three talks. I.M. Sigal spoke about “Effective Dynamics of Magnetic Vortices”. This touched on

Duff's work in the mathematical theory of fluids and described joint work with S. Gustafson of UBC on the dynamics of a quantum-fluid superconductor. This involves constructing a rigorous theory of the time-dependent Ginzburg-Landau equations, and the theory covers as well the related equations of the Abelian gauge Higgs model of elementary particle physics. In geometric terms, these are non-linear equations determining a flow of a connection of a principal $U(1)$ -bundle.

G.A. Elliott, who knew Duff first as a student and later as a colleague, gave “A brief survey of structure and classification theory for amenable C^* -algebras”, reaching from the beginnings of the subject in work of Murray and von Neumann, describing the contributions of Gelfand and Naimark and then the latest developments that begin with Glimm and Bratteli and culminate in the “Elliott program”, a promising attempt to classify the algebras in question through their K -Theory.

L. Nirenberg, who knew Duff from his visits to the Courant Institute, concluded the conference with “Remarks on the distance-to-the-boundary-function and cut locus”, a thoroughly fascinating talk in which he considered a smooth bounded domain on a Riemannian manifold and looked at the distance from any boundary point along the interior normal to the cut locus. He reported on his joint proof that this function is Lipschitz continuous, describing in enjoyable clarity the geometric principles and intuition that enter into the argument.



George F.D. Duff, F.R.S.C.

The Memorial Service

The memorial service that followed the conference featured remembrances by E.P. Mason, a lifelong friend of Duff and his family, and T. Rooney, a colleague of long-standing.

A. Sherk paid tribute to Duff's contributions to the Canadian mathematical community through the Canadian Mathematical Society and N. Derzko fondly recalled Duff's vision in applied mathematics. Duff's wife P. Sauerbrei gave some personal reminiscences. There was as well a message from the current President of the CMS, J. Borwein, who recalled meeting Duff first as a child almost 40 years ago when accompanying his father to CMS summer meetings, and then later at his first job interview with Duff in 1973. He emphasized Duff's contributions through his tireless editorial work on behalf of both the Royal Society of Canada and the CMS.

Ragnar-Olaf Buchweitz (Toronto)

Director's Message *continued from page 1*

the connection to Jones' knot polynomial, but experts in the audience pointed out afterwards that it can be obtained as a special instance of the Tutte polynomial. This is yet another example of how various branches of mathematics are intertwined, and indicates the insight which Bill Tutte brought to his life-long work in graph theory and combinatorics.

Saturday November 3 was the date of a Graduate Information Day. Eight universities, from London to St. Johns, sent representatives to show off their graduate programs in mathematics. Senior undergraduates were invited to attend. Talks by Cam Stewart (Waterloo) and Walter Craig (McMaster) were the main attraction.

On Tuesday November 6, we hosted a special industry event to introduce the Fields Institute to a group of 30 senior people from business and industry. There was lively discussion among the participants, who went away with a new appreciation for Fields and its commercial/industrial mathematics program. We hope to follow up with some more focused activities to deepen the relationships which were begun. This was perhaps long overdue, but we now have an opportunity to build

more activity reaching outside of academia.

As I am writing this, a workshop on Shimura varieties is going on across the hall, with many of the top people in the field participating. There are other ongoing activities as well, such as regular seminars in string theory, non-commutative geometry, finance, and applied math.

If you have research activities in the mathematical sciences which look like they might fit into our broad scheme of activity, don't hesitate to contact me about it.

Looking ahead, there are many things, but I will highlight only one. The Fields Institute is approaching its tenth birthday, and we are planning a special Fields Institute 10th Anniversary Retrospective to celebrate this on June 18 - 19. We are planning an exciting program of plenary talks to reprise many of the successful programs run at Fields over the past decade. This will occur immediately after the CMS meeting in Laval, and we hope that many will take advantage to help us celebrate our first ten years.

Kenneth R. Davidson, Director

Numerical Challenges *continued from page 1*

nized by Tony Humphries from Sussex University in the UK. The second, on Validated Methods for ODEs and DAEs, September 10 to 14, was organized by Ken Jackson from the University of Toronto. Both meetings included lectures by the attendees, but emphasized interaction between the participants during the ample discussion periods. This interaction has already spawned several research collaborations.

At the time of writing (October 12) we are looking forward to three other larger workshops at the Fields Institute this fall:

- Numerical Linear Algebra in Scientific and Engineering Applications, October 29 to November 2;
- Computational Biology, November 19 to 28;
- Computational Challenges in Dynamical Systems, November 29 to December 2.

We are also eagerly awaiting this fall's Coxeter Lecture Series to be given by Gene Golub of Stanford University in conjunction with the Workshop on Numerical Linear Algebra in Scientific and Engineering Applications. (See articles on Coxeter Lecture Series, p. 4, and Workshop on Numerical Linear Algebra, p. 5.)

Our two graduate courses on Numerical Linear Algebra and

on the Numerical Solution of ODEs are underway. The first course is being taught by Christina Christara and the second is being co-taught by Wayne Enright and Ken Jackson. All three instructors are from the University of Toronto. Regrettably the third graduate course on Numerical Solution of SDEs that we had planned for the fall has been postponed. We hope that we will be able to offer it next term.

We have just begun an advanced short-course, consisting of five two-hour lectures on Matrix Valued Function Theory taught by Olavi Nevanlinna of Helsinki University of Technology. We are looking forward to another advanced short-course of similar duration later this fall on Numerical Bifurcation and Centre Manifold Analysis to be taught by Klaus Böhmer of Philipps-Universität Marburg.

In addition, we are enjoying an exciting weekly seminar series with lectures given by the senior research visitors to the Fields Institute and the postdoctoral fellows associated with the program.

For more details about the program, please see our webpage: www.fields.utoronto.ca/programs/scientific/01-02/numerical/.

Ken Jackson (Toronto)

The Fields Institute in Mathematical Sciences publishes the Newsletter three times a year (September, December and June).



Director: Kenneth R. Davidson

Managing Editor: Janet Stern

Deputy Director: Bradd Hart

Publications Coordinator: Elena Kaufman

Scientific Editor: Carl Riehm

Distribution Coordinator: Laura Gass

Coxeter Lecture Series

Gene Golub, Department of Computer Science, Stanford University

This fall's Coxeter Lecture Series was given by Gene Golub, Fletcher Jones Professor of Computer Science at Stanford University.

Golub's major distinctions include the B. Bolzano Gold Medal for Merit in the Field of Mathematical Sciences (1994), Member of the National Academy of Sciences (1993) and of the National Academy of Engineering (1990), SIAM President (1985-87), Director of the Scientific Computing and Computational Mathematics Program at Stanford University (1988-98) and Chairman of the Computer Science Department at Stanford University (1981-84).

Prof. Golub's work in matrix computation devises and analyzes algorithms for solving numerical problems arising in science and statistics. Specifically, he develops algorithms for solving linear systems with special structure, computes eigenvalues of sequences of matrices, and estimates functions of matrices.

Golub's Coxeter Lectures focused on estimates for, and bounds on, the quadratic form $u^T F(A)u/u^T u$, where A is an $n \times n$ symmetric, positive definite matrix, u is a real vector and F is a dif-



Gene Golub

ferentiable function. This problem arises in estimating errors of linear systems, computing a parameter in a least squares problem with a quadratic constraint and bounding elements of the inverse of a matrix.

He described a method based on the theory of moments and numerical quadrature for estimating the quadratic form. A basic tool in this approach is the Lanczos algorithm which can be used for computing the recursive relationship for orthogonal polynomials.

He discussed several extensions of these ideas to problems in statistics and to parameter estimation for solving ill-posed problems. The methods developed are useful in conjunction with Monte Carlo computations.

In addition, methods for updating and downdating recurrences for orthogonal polynomials using modified moments were discussed and some numerical results showing the efficacy of these methods were presented.

For more details about the this fall's Coxeter Lectures, see: www.fields.utoronto.ca/programs/scientific/01-02/numerical/coxeter/golub.html.

Ken Jackson (Toronto)

First Annual McMaster Optimization Conference: Theory and Applications (MOPTA 01)

This conference was sponsored by McMaster University's Faculty of Engineering, The Fields Institute, IBM Canada, and Materials and Manufacturing Ontario, as part of the Fields Special Year on Numerical and Computational Challenges in Science and Engineering. The conference was held on August 2-4, 2001 and was hosted by the Advanced Optimization Lab of the Department of Computing and Software at McMaster.

The Organizing Committee Chair was Tamás Terlaky (Computing and Software, McMaster), and the committee consisted of: Stavros Kolliopoulos (Computing and Software, McMaster), Tom Luo (Electrical and Computer Engineering, McMaster), Jiming Peng (Operations Research, Delft University of Technology), and Henry Wolkowicz (Combinatorics and Optimization, Waterloo).

The conference was attended by 89 paid registrants, 7 invited speakers and 2 guests from IBM Canada and Materials and Manufacturing Ontario. Fifty-seven of the registrants were from Canadian institutions, 17 from the USA and the rest from Asia, Europe and Latin America.

Seven distinguished invited speakers gave one-hour talks on topics of wide interest. They were: Dimitri Bertsekas (MIT), John Dennis (Rice), Ignacio Grossmann (Carnegie Mellon),

Don Jones (General Motors), Michael Todd (Cornell), Lieven Vandenberghe (UCLA) and David Williamson (IBM, Almaden). The technical program was capped by a well-attended banquet at the Royal Hamilton Yacht Club.

The organizing committee received excellent feedback from many of the attendees and is planning to repeat the event this coming summer, again as part of the Fields Special Year on Numerical and Computational Challenges in Science and Engineering.

The conference (www.cas.mcmaster.ca/~mopta/) and the ongoing Optimization seminar series (www.cas.mcmaster.ca/~oplab/seminar/seminar.html) at the Advanced Optimization Lab aim to establish McMaster as one of the hubs for optimization activities in Ontario to foster interaction with researchers in Canada and abroad.

Stavros Kolliopoulos and Tamás Terlaky (McMaster University)



John Dennis and Tamás Terlaky

Second Statistics Workshop at the University of Manitoba

This workshop took place on July 4-5, 2001 at the University of Manitoba. It attracted more than 30 participants and featured 15 talks. It was organized by John Brewster, James C. Fu, Liqun Wang and Ricardas Zitikas of the Department of Statistics and the Institute of Industrial Mathematical Science at the University, and was sponsored by the Fields Institute.

The goal of the workshop was to bring together researchers in the fields in which faculty at the University are active. These fields include the theory and methods of runs and patterns, statistical computation and inference, and industrial statistics.

The meeting opened with a talk by Miklos Csorgo of Carleton University. He surveyed the development of probability theory and stochastic processes, in particular Brownian motion, and the impact of such figures as Einstein, Wiener and Erdos on the subject. Michael Stephens of Simon Fraser then spoke on some problems concerning runs and patterns. His story of a years-long refereeing process elicited considerable sympathy from the audience.

James Fu, one of the organizers, spoke on the finite Markov chain imbedding technique and its application to the calculation of waiting time distributions of runs and patterns. This

technique has attracted a lot of attention in recent years, and one of Fu's papers on the subject recently won the ISI Citation Classic Award for the period 1981 to 1999.

This was followed by talks on the distributions of spacing of order statistics by Mik Bickis (Saskatchewan), on nonparametric precedence testing and alternative procedures by N. Balakrishnan (McMaster), and on convergence of empirical measures for continuous stationary processes by Youri Davydov (Lille).

The second day of the conference had several talks on runs and patterns: exact and approximate distributions of multiple scan statistics by M.V. Koutras (University of Piraeus), distributions of sub-patterns and waiting time distributions of higher dimensional patterns by Katuomi Hirano (Institute of Statistics) and Sigeo Aki (Osaka), and a survey lecture on four schools of methodology on runs and patterns by S.G. Mohanty (McMaster).

Talks were also given by John Braun (Western Ontario), Ming-Ying Leung (Texas), Haijun Li (Washington State), Galit Shmueli (Carnegie Mellon), and Liqun Wang (Manitoba).

The organizers appreciate the sponsorship of the Fields Institute and in particular the kind advice and support of Bradd Hart.

Liqun Wang (Manitoba)

Workshop on Numerical Linear Algebra in Scientific and Engineering Applications

We have just completed a very successful Workshop on Numerical Linear Algebra in Scientific and Engineering Applications (October 29-November 2). In the last decade, computational work has become as important as theoretical and experimental work in many branches of science and engineering. Numerical linear algebra is often at the heart of many computational science and engineering problems, such as materials science simulations, computational finance, structural biology, and image and signal processing, to name just a few. The success of such computational work relies heavily on the development of state-of-the-art algorithms for solving various numerical linear algebra problems.

Just a few of the topics discussed at this workshop include:

- A Recursive Least Squares Approach for GPS Based Positioning;
- Solution of a three-body problem in quantum mechanics

using sparse linear algebra on parallel computers;

- Numerical challenges of large-scale first-principles molecular dynamics;
 - Computational Challenges in Cryo-Electron Microscopy Image Reconstruction;
 - Approximating Eigenpairs in Quantum Chemistry;
 - Techniques for Forward and Inverse Electromagnetic Problems in 3D;
 - Preconditioned Iterative Methods for Image Restoration;
 - Penalty Methods for American Options;
 - Hedging with Value-at-risk.

The excellent facilities at the Fields Institute made for a very enjoyable conference, with ample opportunities for fruitful discussion among the participants.

For more information about this workshop, see the webpage: www.fields.utoronto.ca/programs/scientific/01-02/numerical/linear_algebra/.

Ken Jackson (Toronto)

International Conference on Scientific Computation and Differential Equations (SciCADE)

This conference took place in the Coast Plaza Hotel, Vancouver from July 29-August 3 with about 230 registered attendees from Canada and abroad.

The meeting was sponsored by Fields, PIMS, NSERC and SIAM, and was the opening event of the Fields Institute Thematic Year on Numerical and Computational Challenges in Science and Engineering. The scientific committee members were Uri Ascher (Chair), Georg Bock, Kevin Burrage, Arieh Iserles, Linda Petzold and Bob Russell. The local committee members were Uri Ascher (Chair), Eldad Haber, Bob Russell, Steve Ruuth, Manfred Trummer and Jim Varah.

The conference theme was scientific computing involving the numerical solution of differential equations (ordinary differential equations, partial differential equations, dynamical systems, differential algebraic equations and software.

Plenary speakers included: Lorenz Biegler (CMU), Kevin Burrage (Queensland), Stephen Campbell (North Carolina State), Luca Dieci (Georgia Tech), Leslie Greengard (Courant Institute), Thomas Hou (Caltech), Christian Lubich (Tuebingen), Reinout Quispel (La Trobe), Sebastian Reich (Imperial College, London), Gustaf Soderlind (Lund), and Demetri Terzopoulos (New York and Toronto).

Numerical techniques in applications were emphasized. These included: optimization and optimal control, chemical and mechanical engineering, stochastic differential equations, level-set methods, molecular dynamics, computer graphics and robotics. (Conference website: www.pims.math.ca/scicade/).

Uri Ascher (UBC)

The Fields Institute Mathematics Education Forum, Fall 2001

The Mathematics Education Forum was quickly off the mark this fall with meetings at the end of September and October. Over 20 people attended the two Saturday meetings; minutes for those meetings can be found at the secure website www.fields.utoronto.ca/programs/mathed/meforum/. (The username and password are both *school*). Featured at the September meeting was a lively discussion of potential future topics for Forum meetings. Suggested future speakers included a representative from the Ontario College of Teachers to address qualifications in mathematics for elementary school teachers, a dean of education to talk about how much mathematics fits in their agenda, and someone who could speak about the implementation of technology in the mathematics classroom.

The October meeting saw a discussion led by Gary Flewelling around the article "Choosing content that's worth knowing" by George G. Nelson in the October issue of Educational Leadership, www.ascd.org/frameedlead.html.

Separate discussion groups addressed the question of the type of learning environment necessary in a mathematics classroom to support content that's worth knowing.

Growing out of the work of the task force on the shortage of qualified mathematics teachers in Ontario, the Forum has launched the *Dare to Count* poster campaign in Ontario high schools. The poster, which can be viewed at www.dareto-count.org was sent to the mathematics department head at each high school in Ontario. The link leads to a page that gives detailed information about the process of choosing mathematics as one of your teaching subjects. It is hoped that mathematics teachers and guidance counselors will encourage appropriate students to consider teaching mathematics as a career choice and that this website will help with some of the basic questions about how to do this.

By the time you read this, the task force on Online Mathematics Courses will have held a three-day workshop meant to identify the advantages and disadvantages of on-line instruction. Look forward to a full report in the next newsletter.

FOR UP-TO-DATE INFORMATION ABOUT THE FIELDS INSTITUTE

Please visit our web site at www.fields.utoronto.ca

Write to us at geninfo@fields.utoronto.ca to receive our monthly e-newsletter

Call for proposals, nominations and applications

NEWSletter

Please see the website for detailed information on the proposal and nomination process: www.fields.utoronto.ca/proposals.

THEMATIC PROGRAMS AND GENERAL SCIENTIFIC ACTIVITIES

Proposals for year long or half-year programs at The Fields Institute are currently solicited for the academic years 2003-04 and beyond. We also support programs of one day to one month of concentrated activity in a specific area of current research interest in the mathematical sciences. These activities can be in the form of workshops, seminars, conferences and/or summer schools. *Please see www.fields.utoronto.ca/proposals.*

POSTDOCTORAL OPPORTUNITIES AT THE INSTITUTE

Applications are invited for postdoctoral fellowship positions for the 2002-2003 academic year. Two programs will be offered at the Institute: Set Theory and Analysis (September-December, 2002) and Automorphic forms (January-August, 2003). Qualified candidates who have recently completed a PhD in the mathematical sciences are encouraged to apply. These fellowships provide for a period of at least one year engaged in research and participating in the activities of the Institute. One or more positions may involve teaching or carrying out collaborative research with business or industrial partners. Applicants seeking postdoctoral fellowships funded by other agencies (such as NSERC and international fellowships) are encouraged to request the Fields Institute as their proposed location of tenure, and should apply to the address below for a letter of invitation. Please see www.fields.utoronto.ca/proposals/postdoc.html.

In addition to regular postdoctoral support, the Jerrold E. Marsden Postdoctoral Fellowship will be awarded. It pays a stipend of \$40,000 (Cdn) and provides for a twelve-month period at the Institute for research and participation in the activities of the core program. No teaching is required. In addition to the stipend, a \$2000 (Cdn) research grant will be available during the tenure of the award. Standard NSERC guidelines will apply to this grant.

Postdoctoral applications should reach the Institute by January 2, 2002.

CRM-FIELDS PRIZE

The Centre de recherches mathématiques (CRM) and the Fields Institute for Research in Mathematical Sciences solicit nominations for this joint prize in recognition of exceptional achievement in the mathematical sciences. The deadline is October 15. Previous recipients are H.S.M. Coxeter, George A. Elliott, James Arthur, Robert Moody, Stephen A. Cook, Israel Michael Sigal, and William T. Tutte. Please see www.fields.utoronto.ca/proposals/crm-fields_prize.html for details.

NATIONAL PROGRAM COMMITTEE GRANTS

The three Canadian Institutes in the Mathematical Sciences – CRM, Fields and PIMS – support joint activities in the mathematical sciences through the National Program Committee. Program proposals of national interest lying outside the direct mandate of any of the individual institutes are welcome. Please see www.fields.utoronto.ca/proposals/natprogcomm.html for details.

Please send applications, nominations and proposals to:

The Director
The Fields Institute for Research in Mathematical Sciences
222 College Street, 2nd Floor
Toronto, Ontario
M5T 3J1 Canada

Activities at the Fields Institute

January – August 2002

NEWSletter

Please see www.fields.utoronto.ca/programs/ for detailed information on these activities, or write to us at programs@fields.utoronto.ca.

THEMATIC PROGRAMS

Numerical and Computational Challenges in Science and Engineering. August 2001-August 2002

Organizers: U. Ascher, H. Brunner, T. Chan, T. Coleman, W. Enright, J. Flaherty, P. Forsyth, M. Fortin, A. George, K. Jackson, B. Langford, B. Russell, S. Shen

For information see www.fields.utoronto.ca/programs/scientific/01-02/numerical/ or write to us at numerical@fields.utoronto.ca.

February 4 - 22, 2002 Lecture Series on Numerical and Computational Challenges in Environmental Modelling
Speaker: Z. Zlatev

February 25 - March 1, 2002 Week on Computational Challenges in Mathematical Finance
Organizers: T. Coleman, T. Hurd, P. Forsyth, K. Vetzal

February 25 - 26 Short Course on PDE methods for path-dependent options

February 27 - March 1 Computational Methods and Applications in Finance Workshop

May 13 - 16, 2002 Workshop on Numerical Challenges in Computer Animation
Organizer: D. Terzopolous

May 26 - June 1, 2002 Informal Working Group on Validated Methods for Optimization
Organizers: G. Corliss, T. Csendes, K. Jackson, B. Kearfott

July 13 - 19, 2002 Symbolic Computational Algebra 2002
Fields Institute special meeting on Symbolic and Numeric Computation in Geometry, Algebra and Analysis, held at University of Western Ontario
Organizers: R. Corless, E. Green, S. Hosten, R. Laubenbacher, V. Powers, G. Reid
For information see www.orcca.on.ca/sca2002/

August 6 - 9, 2002 IMACS International Conference on Adaptive Methods for PDEs

August 12 - 15, 2002 The 2002 Workshop on the Solution of Partial Differential Equations on the Sphere
Organizers: J. Drake, P. Swarztrauber, K. Jackson, D. Williamson

Coxeter Lecture Series
March 4 - 15, 2002 "Solving Wave Propagation Problems in Heterogeneous Media"
Speaker: R. J. LeVeque, Applied Mathematics Department (Washington),

Graduate Courses
Winter 2002 Numerical Solution of PDEs
Numerical Solution of Optimization Problems

Summer Course
July 29 - August 2, 2002 Numerical Solution of Advection-Diffusion-Reaction Equations
Instructor: J. Verwer

GENERAL SCIENTIFIC ACTIVITIES

June 18- 19, 2002 Fields Institute 10th Anniversary Retrospective
For information, see www.fields.utoronto.ca/programs/scientific/01-02/fields10anniversary.

July 15 - August 10, 2002 International Conference on Representation Theory of Algebras and Related Topics (ICRA X)
Organizing Committee: S. Berman, Y. Billig, R.-O. Buchweitz, V. Dlab, E. Neher, S. Liu
For information, see www.fields.utoronto.ca/programs/scientific/02-03/ICRA-X/.

July 22 - August 16, 2002 1st AARMS Summer School at Memorial University
Organizer: E. Goodaire For information, see www.math.mun.ca/~aarms/SS2002.

August 7-11, 2002 Workshop on Geometry, Dynamics, and Mechanics in Honour of the 60th Birthday of J.E. Marsden
Organizing Committee: A. Bloch, P. Newton, T. Ratiu, S. Shkoller, A. Weinstein
For information, see www.fields.utoronto.ca/programs/scientific/02-03/marsden_workshop/

The Early History of the Fields Medal

Although all mathematicians know about the Fields Medal, surprisingly few, even among Canadians, are aware of its Canadian provenance. This is probably less true now than it was a few years ago – possibly because of the arrival of The Fields Institute on the scene? In any case the history of the Fields Medal is less well-known, and so we offer this brief account. A longer version can be found in the article by Henry Tropp mentioned at the end of this sketch.

John Charles Fields

Fields was born in Hamilton, Ontario, then Upper Canada, in 1864. His father operated a leather shop at 32 King St. West, and the family lived nearby at 150 King St. East. (Both of these buildings have long since disappeared – the site of the shop is now occupied by Jackson Square, a shopping complex, and that of the house by the Ramada Inn). Fields graduated from the University of Toronto in 1884, and then left to study at Johns Hopkins University. That institution apparently was the only North American University which stressed research at that time. Its mathematics program had been set up by J.J. Sylvester. Fields was awarded a Ph.D. by Johns Hopkins in 1887. His thesis was entitled *Symbolic Finite Solutions and Solutions by Definite Integrals of the Equation $d^n y/dx^n = x^m y$* , and was published in the American Journal of Mathematics in 1886. After teaching at Johns Hopkins for two years, he left for Allegheny College in northwestern Pennsylvania.

Fields was understandably dissatisfied with the state of mathematics in North America at that time, and in 1891 he departed for Europe to spend the next 10 years there, combining a modest inheritance from his parents with economical living habits.

Fields's years in Europe, mainly in Berlin but also in Göttingen and Paris, influenced him deeply and reinforced his convictions about the importance of mathematical research. He mingled with many of the greatest mathematicians of that time – Klein, Frobenius, Weierstrass, Fuchs, Hensel – and changed his mathematical interests to algebraic functions. He also developed there a life-long friendship with the Swedish mathematician Gustaf Mittag-Leffler. The persistent rumor that Nobel did not establish a prize in mathematics because Mittag-Leffler had an affair with his wife is certainly incorrect. Nobel never married. But the other version of this rumor, based on hostility between Nobel and Mittag-Leffler, was evidently true – according to a letter from J.L. Synge to Tropp, Fields told Synge that this *was* the case, and Synge remarks that he later confirmed it himself in Sweden. One wonders if this fact and the friendship between Fields and Mittag-Leffler eventually led Fields to establish his award.

Fields returned to Canada in 1902 as a special lecturer at the University of Toronto. He was promoted to associate professor in 1905, professor in 1914, research professor in 1923, and became a Fellow of the Royal Society of Canada in 1909 and of the Royal Society of London in 1913. He spent much of his leisure time in Europe, and it is claimed that he was a personal acquaintance of several reigning monarchs. He attended a dinner party in 1912 given by the King of Sweden.

Fields worked tirelessly to promote mathematical research. Soon after his return from Europe, he lobbied the Ontario Legislature for support for research. He persuaded the government to provide to the University of Toronto a special annual research grant of \$75,000, a significant sum at a time when professors earned less than \$1,000 per year.

He also devoted his efforts to the establishment of the National Research Council (from which N.S.E.R.C. later developed) and the Ontario Research Foundation.

The Royal Canadian Institute was another of his endeavors. He served as its President between 1919 and 1925, and attempted to transform it into an instrument for the promulgation of scientific thought as well as a centre for actual research. To that end, he spent much time and money in persuading distinguished scientists to lecture to the membership of the Institute and the public – the Saturday evening lectures became very popular during his tenure of office. Fields's vision of the Royal Canadian Institute as a centre of research did not materialize, but we would like to think that the Fields Institute is a worthy realization of his dreams.

The International Mathematical Congresses

The international mathematics community has held International Mathematical Congresses every four years since 1897. The first of these meetings was held in Zürich, but they were soon interrupted by World War I. They resumed in 1920 with a meeting in Strasbourg at which mathematicians from the “Central Powers” (Germany, Austro-Hungary, Bulgaria and Turkey) were excluded. There was a great deal of controversy about whether or not they would be allowed to attend the next meeting in 1924 in New York. The American Mathematical Society withdrew its support for that meeting shortly thereafter because of difficulty in gaining financial backing for it. Through Fields's efforts, Toronto was chosen in 1922 as the venue of the 1924 Congress. He evidently had mixed feelings about holding the meeting with the Central Powers excluded, but decided that it was important to hold the Congress in any case. He became its President and worked steadily during the next two years to ensure its success, far from obvious at the time. It was not until 1928 in Bologna that mathematicians from the Central Powers rejoined the international community. When the German delegation, led by David Hilbert, then an old man, entered the hall, the audience rose and cheered wildly. As Hilbert said of the occasion, “All limits, especially national ones, are contrary to the nature of mathematics.”

The International Mathematics Union was initially formed in 1923 to support the Congresses and international collaboration in mathematics. But it was not truly representative of the mathematical community at that time due to its exclusionary policies. An attempt in the 1930s to revive the IMU failed because of the impending war. The IMU was reborn in 1950, and finally in 1962 came to play its present role through its involvement in the structural details of the Stockholm Congress.

The 1924 Toronto Congress was very successful, and was followed by an organized rail excursion to British Columbia accompanied by Fields. For many nights he got no sleep, and on his

return to Toronto his health broke down. From that time on, he never regained the vigor of his former years. He did manage to complete the proceedings of the Congress with the help of his colleague J. Chapelon. They appeared in two large volumes in 1928.

The Fields Medal

The history of the Fields Medal itself begins in the Committee of the International Congress set up by the University of Toronto in November of 1923. Fields was the chairman and Synge the secretary. Although Fields probably conceived of such an award at some earlier time, the first recorded mention of it is in the minutes of February 24, 1931 where it is “resolved that the sum of \$2,500 should be set apart for two medals to be awarded in connection with successive International Mathematical Congresses through an international committee appointed for such purpose initially by the executive of the International Mathematical Congress, but later by the International Mathematical Union”. The \$2,500 was evidently the balance on hand after all expenses of the 1924 Congress had been met. At the next meeting of the committee, in January 1932, Fields indicated that the idea of the medal had the support of the major mathematical societies of France, Germany, Switzerland, Italy and the U.S.

At this same meeting, he outlined the principles behind the proposed medal. The genesis of the rule that it be only awarded to mathematicians under the age of forty is evidently the statement that “... while it was in recognition of work already done, it was at the same time intended to be an encouragement for further achievement on the part of the recipients and a stimulus to renewed effort on the part of others”. Then he continues “In commenting on the work of the medalists it might be well to be conservative in one’s statements, to avoid invidious comparisons explicit or implied. The Committee might ease matters by saying that they had decided to make the awards along certain lines not alone because of the outstanding character of the achievement but also with a view to encouraging further development along these lines”. And mindful of the turmoil of ten years earlier, he added “the medals should be of a character as purely international and impersonal as possible. There should not be attached to them in any way the name of any country, institution or person”. Of course in spite of Fields’s intentions, it became known as the Fields Medal when it was awarded for the first time in Oslo in 1936. It is interesting to note that, at the same meeting, it was decided that “the Chairman should see the Prime Minister of Canada to arrange if possible how permanence of capital and of interest of the fund might be assured” since at this moment (Fall, 2001), there is a movement by The Fields Institute and the Canadian Mathematical Society to increase the meager prize money accompanying the award of the Fields Medal.

Fields then proceeded with the planning of the award of the first medals, but fell ill in May of 1932 and died in August. Just before his death, he called Synge to his bedside, and with a lawyer present, made his will. It included an amount of \$47,000 to be added to the funds for the medal.

Synge carried Fields’s proposal to the Congress in Zürich in September of that year. It was accepted and a committee consisting of Birkhoff, Carathéodory, Cartan, Severi and Takagi was formed to make the first awards at the Oslo Congress in 1936. They chose

Lars Ahlfors of Finland and Jesse Douglas of the U.S.A. Unfortunately war again intervened, and the next ICM was held in Cambridge Massachusetts in 1950, when Laurent Schwartz of France and Atle Selberg of the U.S.A. were selected as Fields Medalists. A list of all Fields Medal winners (with a short description of their work) can be found at www.emis.math.ca/EMIS/mirror/IMU/medals/, and an analysis by Michael Monastyrsky of the effect of Fields Medalists on 20th century mathematics and physics, delivered in a lecture at the Fields symposium “The legacy of John Charles Fields” held in Toronto in June, 2000, is available at www.fields.utoronto.ca/aboutus/FieldsMedal_Monastyrsky.pdf.



The Fields Medal

The medal itself

Fields specified that the medals should “each contain at least 200 dollars worth of gold and be of a fair size, probably 7.5 centimetres in diameter. Because of their international character the language to be employed it would seem should be Latin or Greek”. The medal does in fact meet these specifications (in 1933 dollars!). Its monetary value has at least on one occasion been of critical importance: in the turmoil at the end of World War II, Ahlfors was separated from his wife, and was only allowed to leave Finland with 10 crowns. He smuggled out his Fields Medal and pawned it, enabling him to reach his wife in Zürich. (He later retrieved it with the help of some Swiss friends).

The medal is struck every four years in the Royal Canadian Mint, and was designed by the Canadian sculptor R. Tait McKenzie. He chose a picture of Archimedes for the obverse from a collection at Columbia University. The Latin inscription from the Roman poet Manilius surrounding his image may be translated “To transcend one’s human limitations and master the universe”. The inscription on the reverse may be translated “Mathematicians gathered together from the whole world honor noteworthy contributions to knowledge”. Behind the inscription are a laurel branch, and a diagram of a sphere contained in a cylinder copied from an engraving on Archimedes’ tomb.

This article has been written with information from several sources including the article by Henry S. Tropp, *The origins and history of the Fields Medal*, *Historia Mathematica* 3 (1976), 167-181, an article in *Saturday Night* published in 1934 announcing the launching of the Fields Medal, an obituary of Fields by Synge in the *Journal of the London Mathematical Society* in 1933, a lecture by T. Archibald at the *Symposium on the Legacy of John Charles Fields* in June, 2000 at the Fields Institute, and unpublished articles by Frances Hoffman and Elaine Riehm (who was also responsible for the adoption of the name “Fields Institute” in 1987). I would also like to thank Florin Diacu, Allyn Jackson and Robert Osserman for their help.

Carl Riehm (Fields Institute)

The Individual Finance and Insurance Decisions (IFID) Conference

The Fields Institute and the Individual Finance and Insurance Decisions (IFID) Centre jointly sponsored and organized a full-day conference devoted to the topic of Personal Risk Management (September 24, 2001). The conference organizer was Moshe Milevsky, an Associate Professor of Finance at York University, and the Executive Director of The IFID Centre.

The organising committee consisted of Moshe Milevsky, Bradd Hart (Fields), Tom Salisbury (York) and Ted Cadsby (CIBC).

While corporate financial risk management has attracted extensive attention over the last few decades, personal financial risk management has not. This area is potentially more challenging since the risks we face in our daily life are more diverse, involving not only our financial wealth, but our human capital, real estate and health status.

The Fields Institute invited some of the leading research experts and practitioners in the field to share their insights. The audience heard an inspiring keynote address from Richard Roll (UCLA) who spoke about the general topic of risk and return, as well as his unique model for managing equity portfolios. He reminded the audience about the large amount of trading data that is needed to ascertain if certain markets or strategies are performing better than others. Roll is well known for his contributions, together with Steve Ross (MIT), to finance and portfolio theory.

Zvi Bodie (Boston U.) spoke about retirement investment strategies and some innovative savings vehicles such as iBonds and other derivative-based strategies that should be part of the retirement savings equation. Bodie is well known for his joint work with the Nobel Laureate Robert Merton and has authored many papers and books in the field of pension finance and economics.

Mike Orszag (City College and Watson Wyatt, UK) spoke

about immediate annuities and their place in the optimal portfolio of older people. He provided empirical and theoretical evidence that immediate annuities provide superior investment returns at higher ages.

Pauline Shum (York) presented her paper “Portfolio Choice in the Presence of Personal Illiquid Projects” which is forthcoming in the *Journal of Finance*. She discussed the puzzle of why so many young people invest such a large portion of their financial wealth in risk-free instruments, as opposed to the equity-based products predicted by lifecycle theory. She described data from the Survey of Consumer Finances and talked about the importance of illiquid commitments as they pertain to portfolio theory.

Ted Cadsby (CIBC) gave a humorous and insightful view of working with individual investors and consumers on the front line. Ted is a well-known Canadian commentator and public persona who has met with thousands of individual investors over the years. He pointed out many of the puzzling behavioural anomalies that persist in their daily patterns. His message about the superiority of index-funds and passive investment vehicles was reassuring to many in the audience.

Robert Brown (Waterloo), the President of the Society of Actuaries, spoke about the demographic impact of public pensions and social programs on the individual investor. He presented a macro-economic model of retirement – which induced a lively debate about tax incentives in Canada – and the proper way to reform Social Security in the United States.

Finally, the audience heard from Larry Kotlikoff (Boston U.), a well-known tax and macro-economic policy expert in the United States. He discussed a revolutionary financial software program that he has developed, called ESPlanner – Economic Security Planner. The main distinguishing feature of his framework is the ability to incorporate the Modigliani lifecycle hypothesis and optimal insurance purchases into a parsimonious model aimed at financial planners.

All in all, the conference was quite successful, and The IFID Centre is hoping to turn this into an annual event.

Moshe Milevsky (IFID)

GRADUATE INFORMATION DAY

On the afternoon of Saturday November 3 we held a Graduate Information Day at The Fields Institute. Eight universities, McMaster, Memorial, Ottawa, Queens, Toronto, Waterloo, Western, and York, had pamphlets and posters to advertise their graduate programs in the mathematical sciences. Students came from many of the surrounding universities.

The main attractions were two interesting talks aimed at senior undergraduates. Cam Stewart (Waterloo) spoke on the “abc Conjecture” showing how an apparently innocent and peculiar looking conjecture in fact implies many amazing and wonderful consequences about the relationship between the additive structure of integers and their multiplicative structure

(primes). Walter Craig (McMaster) talked about the mathematics of water waves, with many interesting slides showing the experimental and numerical side of the subject even though he sees himself on the purer side of the fence. He showed how the PDEs arise from the physics, and talked about various approaches to their analysis.

Afterwards, everyone enjoyed a reception and wandered around talking to the representatives from each department about their programs. Overall it was a success and we look forward to making this an annual event.

Kenneth R. Davidson, Director

News From AARMS

The Atlantic Association for Research in the Mathematical Sciences, an affiliate member of The Fields Institute, comprises all universities in Atlantic Canada. Hermann Brunner of Memorial University of Newfoundland is the director, and its sponsoring universities are Dalhousie, Memorial and UNB. Most of its funding has come from the National Program Committee (the CRM, Fields and PIMS). Further details about AARMS can be found at www.math.mun.ca/~aarms/.

Some of its recent and future events are:

- Workshop on *Groups, Rings, Lie and Hopf Algebras* at Memorial University, May 28-June 1, 2001. This event, funded by the NPC, was organized by Yuri Bahturin, Edgar Goodaire, Mike Parmenter and Yiqiang Zhou of Memorial. It attracted some 40 researchers and graduate students from 10 countries, including Efim Zelmanov (Yale), a 1994 Fields Medalist. Zelmanov also gave a well-attended public lecture entitled "Abstract algebra in the 20th century", and supported by the President of Memorial University. Susan Montgomery (University of Southern California) was designated as the AARMS Distinguished Lecturer for this workshop.
- Workshop on *Dynamical Systems and Their Applications in Biology* in Cape Breton, NS, August 2-6, 2001. This NPC funded workshop was organized by Shigui Ruan (Dalhousie), Gail Wolkowicz (McMaster) and Jianhong Wu (York). Close to 40 talks by participants from nine countries were presented. The AARMS Distinguished Lecturer was Hal Smith (Arizona State University).
- Workshop on *Modeling and Scientific Computation* at the University of New Brunswick, September 28-30, 2001. The aim of this very successful workshop, organized by Viqar Husain, John Stockie and James Watmough (UNB) and partially funded by the NPC, was to expose mathematical modelers and computational scientists in the areas of biology, finance, fluid dynamics, and relativistic physics, to state-of-the-art research and computational techniques and to questions of interdisciplinary research. There were four plenary lectures, and the workshop was attended by some 40 researchers and graduate students. This AARMS event was part of the Fields Institute's Thematic Year on Numerical and Computational Challenges in Science and Engineering (www.math.unb.ca/~masc01/).

- The *APICS 2001 Meeting* at St. Francis Xavier University, Antigonish, NS, October 19-21, 2001. In addition to its educational component consisting of an undergraduate student competition and paper sessions, this year's mathematics meeting of APICS (see also its homepage: www.math.mun.ca/~apics/), featured three AARMS special sessions:

- the *East Coast Discrete Mathematics Conference*, organized by Ping Wang (St. Francis Xavier) and funded by The Fields Institute;
- the *Atlantic Provinces General Relativity and Cosmology Conference*, organized by Robert van den Hoogen (St. Francis Xavier);
- *Numerical Analysis/Scientific Computing and Computational Applied Mathematics*, organized by Paul Muir (St. Mary's University, Pat Keast and Ray Spiteri, Dalhousie).

A total of thirty talks were presented in the three sessions.

- The first *AARMS Summer School* at Memorial University from July 22 through August 16, 2002. Organized by Edgar Goodaire (Memorial), it is intended for graduate students (and promising advanced undergraduates) from Canada and abroad. It will offer courses in Algebra, Fractal Geometry, Graph Theory, and Numerical Analysis, taught by mathematicians from around the world (see the website, www.math.mun.ca/~aarms/SS2002/, for more details). It is modeled after the highly successful Perugia summer schools. One of our principal goals is to broaden the education of graduate students in our region by bringing them into contact with their colleagues from different educational settings. The initial response from potential participants in Canada, the USA, and many countries in Europe and Asia has been very enthusiastic.
- The Workshop on *Combinatorial Methods in Polynomial Identity Theory* at Memorial University of Newfoundland. This workshop will take place during one week in August/September 2002, and will be the latest in a series of conferences held annually since 1992. It is being organized by Yuri Bahturin (Memorial) and others from Canada, Italy, Israel, Russia and the USA.
- *The 31st Annual Canadian Operator Theory and Operator Algebras Conference* at the University of New Brunswick from May 20-24, 2003. The organizer is Dan Kucerovsky (UNB).

Hermann Brunner (Memorial University)

THE FIELDS INSTITUTE 10TH ANNIVERSARY RETROSPECTIVE

There will be an exciting program of plenary talks reprising some of the programs of the past decade.

Please mark these dates: **June 18-19, 2002.**

For updated information as the date nears, visit our web site at

www.fields.utoronto.ca/programs/scientific/01-02/fields10anniversary.

2001 CRM-Fields Prize Lecturer

William T. Tutte

The CRM-Fields Prize was awarded this year to Professor William T. Tutte of the University of Waterloo. It was presented to him in a ceremony on October 25 at the Fields Institute by the Director, Kenneth R. Davidson. Tutte then delivered a lecture entitled "Sixty Years in the Nets" to the assembled audience.

Rather than attempt to survey his life in research as a whole, Tutte chose to talk about one thread of his work which began while he was an undergraduate chemistry student at Cambridge. The beginning of this thread was the solution of a simply stated problem of recreational mathematics: dissect a square into smaller squares, no two of which are of the same size. Tutte had been introduced to graph theory in high school through reading the book "Mathematical Recreations and Essays" by W.W. Rouse Ball, and he and three fellow students found a way to associate a graph to dissections of a square into smaller squares. This association in fact gives rise to a one-to-one correspondence between certain kinds of graphs, which can be interpreted as electrical circuits, and dissections of rectangles. They found a way to use this association, with the help of Kirchoff's Laws of circuit theory, to solve the original problem of dissecting a square into unequal smaller squares. This involved the "complexity" $C(G)$ of a graph, the number of spanning trees of the graph G . In carrying this out, they developed a technique in circuit theory, the "sum of tree-products" method, that became - and still is - the standard algorithm for computing the impedances of electrical networks.

Tutte's career as a student was interrupted by World War II, during which he worked on decoding at Bletchley Park (cf. the Fields June newsletter). After the war he returned to Cambridge as a PhD student in mathematics and once again took up matters relating to $C(G)$. This function satisfies the recursion relationship $C(G) = C(G_A') + C(G_A'')$ for any edge A . Here $C(G_A')$ is the complexity of the graph G with A deleted and $C(G_A'')$ is that of the graph obtained from G by contracting A to a vertex. Tutte noticed that other graph-theoretic functions, such as the chromatic polynomial $P(G, \lambda)$, satisfy the same recursion relationship (sometimes a suitable sign change must be introduced). $P(G, \lambda)$ is the number of ways of colouring a graph (with each vertex of a different colour than its neighbours) with λ different colours.

A function F satisfying

$$F(G) = F(G_A') + F(G_A'')$$

is called a W-function. It is called a V-function if it has the additional property that it is multiplicative over the connected

components of G . In 1947, Tutte proved the existence of a V-function which specializes in a cubic graph to one useful in finding its 1-factors, and in 1980, gave a characterization of that V-function in the first issue of the European Journal of Combinatorics.

The most general W-function of a graph that he found is now called the "Tutte polynomial" $\chi(G, x, y)$. This now famous polynomial - of which the more recently defined Jones polynomial of knot theory is a special case - was the exclusive subject of a major conference held last summer in Barcelona.

The recursion relationship is also used in the calculation of chromatic polynomials, and another of Tutte's interests in this circle of ideas was the calculation of the zeros of the chromatic polynomial $P(G, \lambda)$. One of his correspondents Dick Wick Hall and two coauthors calculated over 900 examples of $P(G, \lambda)$ for triangulations of the plane which lead to the truncated icosahedron. Gerry Berman calculated the zeros of all of these polynomials. Tutte then noticed that almost all of them had a zero close to

$1 + \tau$ where τ is the "Golden Ratio" $\frac{1}{2}(1 + \sqrt{5})$ and subsequently proved the beautiful theorem that if T is a planar triangulation with k vertices, then

$$|P(T, 1 + \tau)| \approx \tau^{5-k}.$$



"Prof. Tutte: Sixty Years in the Nets"



Ken Davidson presenting the CRM-Fields Prize to William T. Tutte

Tutte brought his lecture to a close by describing other values of λ at which $P(G, \lambda)$ satisfies a special recursion: these are the Beraha numbers $B_n = 2 + 2\cos(2\pi/n)$, $n = 1, 2, \dots$ of which $B_5 = 1 + \tau$.

The lecture was followed by a reception in the Institute atrium.

A biographical sketch of Professor Tutte can be found in the Fields June newsletter of this year on our website at www.fields.utoronto.ca/aboutus/annual_reports/June_2001_Newsletter.pdf.

Carl Riehm (Fields Institute)

A Brief History of Operator Algebras

The theory of operator algebras began in the early part of the twentieth century at essentially the same time as quantum mechanics, and is in fact closely related to that subject. However the larger part of the development of operator algebra theory has occurred much more recently.

Almost seventy-five years ago, John von Neumann axiomatized Hilbert space, and proved the bicommutant theorem for strong operator closed, self-adjoint algebras of bounded linear operators acting in this space – only shortly after Heisenberg’s matrix mechanics, and the equivalent wave mechanics of Schrödinger. Soon afterwards, von Neumann formalized the relation between the two subjects with his monograph, *The Mathematical Foundations of Quantum Mechanics*. At the same time, he vigorously pursued the development of the theory of these algebras of operators (now called von Neumann algebras). This work, which grew to be monumental in scope, was carried out to a considerable extent in collaboration with F.J. Murray. Another collaborator, I. Halperin, worked with von Neumann on the geometrical aspects of operator algebras.

About sixty years ago, I.M. Gelfand and M.A. Naimark drew attention to a phenomenon quite outside the sphere of von Neumann algebra theory. A von Neumann algebra, while it may be considered abstractly if desired, has in a rather strong sense a unique Hilbert space representation. If a self-adjoint algebra of Hilbert space operators is assumed only to be norm-closed – what is now called a concrete C^* -algebra – then in general it has many other, quite different, Hilbert space representations. In quantum mechanical systems of finitely many degrees of freedom, this does not happen. This is illustrated by the unitary equivalence of the Heisenberg and Schrödinger formulations of quantum mechanics, which may be thought of as different, but equivalent, representations of the C^* -algebra of compact operators. While this C^* -algebra has an essentially unique Hilbert space representation, it was proved in the early 1950s by Gårding and Wightman that the C^* -algebra of a system with infinitely many degrees of freedom has infinitely many inequivalent irreducible representations. This had been recognized implicitly ten years earlier by Gelfand and Naimark in their highly non-trivial axiomatization of abstract C^* -algebras.

Although von Neumann algebras have a much simpler representation theory than C^* -algebras, their internal structure is just as complicated. This was a direct reflection of the subtlety of C^* -algebra representation theory. Typically, as soon as the C^* -algebra is not the algebra of compact operators, not only are different representations unitarily inequivalent, but also they may generate non-isomorphic von Neumann algebras.

Two of the unifying themes of operator algebras, since the beginning of the subject, have been the questions of classification up to isomorphism – respectively, for von Neumann algebras and for C^* -algebras. In the presence of natural countability and amenability assumptions, the classification of von Neumann algebras has now been solved, and the classification of C^* -algebras is approaching resolution.

The von Neumann algebra classification question was considered by Murray and von Neumann already in 1935, but only solved about twenty years ago by work of Connes and others building on somewhat earlier major advances – notably the work of Tomita and Takesaki (and others) which showed that a von Neumann algebra may be thought of as evolving in a natural way in time.

The C^* -algebra classification question came up in the commutative case in the work of Gelfand and Naimark in 1943, but the first step after that was the case of infinite tensor products of matrix algebras considered by Glimm in 1959. An extension of Glimm’s results to the more general class of approximately finite dimensional C^* -algebras (AF algebras), in three stages, by Bratteli, me, and Effros, Handelmann and Shen, was completed only in 1979.

My work, for about thirty years, has mainly concerned this C^* -algebra classification question. This has in particular involved the development of operator algebra K-theory. K-theory was present, to be sure, in an embryonic way in the work of Murray and von Neumann. It was the basis of their so-called type classification. In geometry and topology, K-theory was developed into a powerful tool by Grothendieck, Atiyah and Hirzebruch, and Bott’s remarkable periodicity theorem. This theory was recognized as important in operator algebras, partly through my work on AF-algebras.

This expansion of operator algebra theory led to the non-commutative geometry of Alain Connes, which is related in a deep way to many branches of both mathematics and physics. In particular, the invariant used for the von Neumann algebra classification, the so-called Connes-Takesaki flow of weights, may be thought of as K-theoretical in nature. Also the Jones index, basic to the classification of subalgebras of von Neumann algebras (subfactors), and to the discovery of the Jones knot polynomial, is a K-theoretical quantity. Connes’s non-commutative generalization of the Chern character is fundamental in index theory, and in recent work of Connes and Kreimer, has proved to be related to the renormalization problem for Feynman diagrams. Much of string theory as well as M-theory is related to the ideas of Jones and of Connes. And Connes’s non-commutative geometry has provided a possible approach to the Riemann hypothesis.

George Elliott (Toronto)

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